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Synthesis and implementation of λ-approach of slide control in heat-consumption system

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Abstract
The paper proposes an essentially new approach to synthesis and implementation of dynamic objects with three-position relay control. The approach consists in organization of differentiation procedure on the relay element involved into feedback. We considered synthesis of the relay element feedback in tasks of robust and time optimal control of heat-consumption systems. To demonstrate the effectiveness of the proposed approach, a comparative assessment of the results of modeling heat consumption systems with three-position relay control and a traditional linear–quadratic regulator is presented. We attached transient processes plots of active heat-consumption systems which confirm the effectiveness of the synthesized relay control.

Keywords
heat-consumption systems, lambda-regulator, sliding mode, time optimal control, three-position relay control

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Синтез и реализация λ-подхода скользящего управления в системе теплопотребления

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Аннотация
Предмет исследования. Предложен новый подход к синтезу и реализации трехпозиционного релейного управления сложными динамическими объектами. Рассмотрена методика синтеза обратной связи релейного элемента в задачах робастного и оптимального по быстродействию управления системами теплопотребления. **Методы.** Представленный подход заключается в организации процедуры дифференцирования на релейном элементе, включенным в обратную связь системы управления. **Основные результаты.** Для демонстрации эффективности подхода выполнена сравнительная оценка результатов моделирования систем теплопотребления с релейным трехпозиционным управлением и традиционным линейно-квадратичным регулятором. Показаны графики переходных процессов действующих систем, которые подтверждают эффективность синтезированного

релейного управления. **Практическая значимость.** Результаты работы найдут применение при разработке алгоритмов управления системами теплопотребления зданий и сооружений.

Ключевые слова
системы теплопотребления, лямбда-регулятор, скользящий режим, оптимальное по быстродействию управление, трехпозиционное релейное управление

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Introduction

The article considers the problem of synthesis of relay control for a three-position valve of a heat consumption system. Traditionally, such systems use a classic proportional-integral-derivative (PID) controller together with a pulse-width modulator for three-position control of the actuator. When solving problems of improving the quality of control of various non-stationary and non-linear objects, there are issues of practical application of modern methods for the synthesis of such controllers where estimates of the derivatives of the measured state variables are used. This is due not only to the presence of various noises and interferences in the measuring channels, but also to the insufficient bit grid of analog-to-digital converters [1].

During the operation of many control objects, including heat consumption systems, their parameters can change significantly. The use of regulators with robust properties [2] allows, in many cases, to provide the required quality indicators. An important place in the quality criteria is occupied by the requirement for the maximum speed of the control system, which can be performed on the basis of relay controllers.

In the scientific literature, you can find solutions where relay control is synthesized using modern approaches. For example, in works [3, 4], variable structures are used to form a relay law, however, such solutions are aimed at a certain type of control object. Robust properties can also be provided by applying sliding control [5–8]. The most studied are control systems for objects of the 2nd order [9, 10] where the switching line is determined on the phase trajectory [11, 12]. For objects of a higher order with a delay, it is possible to use proven solutions for 2nd order control objects, but the problem of chaterring inevitably arises, the studies of which are devoted to the works [13–15]. A good result can be achieved by generating multi-level control values [16] or quasi-sliding modes with a dead zone [17]. However, such solutions are more limited by the complexity of calculating the derivative of the measured state variable of the control object with discretization errors by level.

Of particular interest are variants of optimal control in terms of speed [18] where the switching line is organized in the phase space [17, 19]. Therefore, it can be assumed that there is a generalized solution for finding the sliding line [20] in a simpler way which has the robust properties of the sliding mode, and, at the same time, provides optimal control in terms of speed. In the paper, the authors propose a similar generalized approach that allows synthesizing a three-position control without estimating the derivative of the measured object state variable.

Generalized solution of optimal and robust control

The linearized mathematical model for equilibrium states of such objects in operator form relative to the position of control valve stem can be written in the following way [21]:

$$W(s) = \frac{k_o(t, x)}{T_v s} \times \frac{\exp(-\tau(t, s)s)}{1 + T_o(t, x)s}, \qquad (1)$$

where T_v is duration of the valve rod shift; s is Laplace transformation operator. The duration $\tau(t, s)$ of transport delay, coefficient $k_o(t, x)$ of transmission and object time $T_o(t, x)$ have a complex dependence on the variables state $x(t)$, and it is ordinarily unsteady.

For this class of objects there is some problem of using of classic methods for optimal control synthesis in construction of regulators based on library modules of a programmable logic controller (PLC).

As an alternative direction, let us consider methods of robust control in sliding mode and time optimal control. Taking as a base the research results of robust control systems in sliding mode [9], we can use the condition of attraction to the sliding trajectory in boundary modes and assume the existence of time optimal conditions. Thus, the object (1) without delay is a special case of mathematical model:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -h(x, t)x_2 + g(x, t)u(t), \end{cases} \qquad (2)$$

where $g(x, t)$ and $h(x, t)$ are positively defined parameters of the objects. For this model there is control

$$u(t) = -sign(a_1 x_1(t) + x_2(t)), \qquad (3)$$

where a_1 is a parameter which determines the sliding trajectory. We know the condition

$$\left|\frac{a_1|x_2| + h(x)}{g(h)}\right| \le 1, \qquad (4)$$

which provides attraction to the sliding trajectory [21].

Fig. 1 shows the cases when the sliding trajectory parameter satisfies the attraction condition (4) when the trajectory of the object in the phase plane is reached again [5, 11, 12].

Such implementation of the controller with variable structure corresponds to the optimal regulation of the object of the second order (1), based on the assumption that the second intersection of the sliding trajectory is provided at the small field near by equilibrium of the system [3, 6, 7, 16]. It should be noted that in conditions of noise

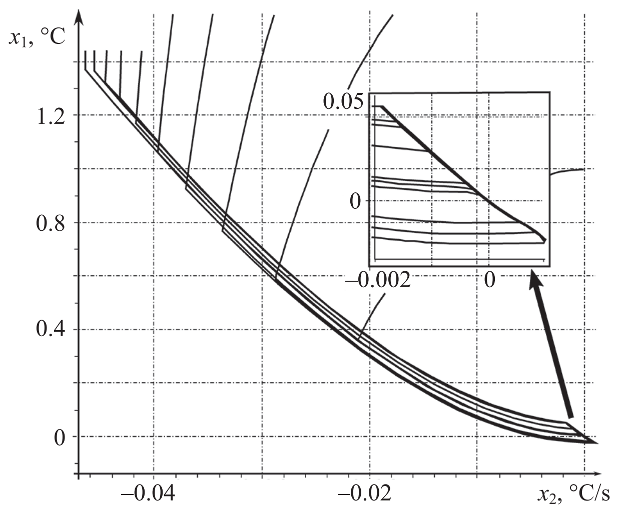

Fig. 1. Time optimal trajectories for management with the fragment of sliding mode

and discretization errors in the measuring channel, the implementation of the sliding mode becomes virtually impossible. This fact is proved by lack of relevant functional and program blocks in the PLC libraries.

At the same time there are known theoretical approaches allowing to perform differentiation on the relay element with integrating feedback without strict requirements for measurement quality [18]. Software implementation of the relay element with feedback makes good preconditions for implementation of sliding modes in control systems implemented on PLC base. This direction of research has significant resource of development [4].

Synthesis of the relay element feedback

Let us bring an additional state variable $x_3(t)$ physically connected with variables $x_1(t)$ and $x_2(t)$ through the control u into the mathematical model (2) of the object of the second order. Further, to calculate the control instead of (3) we will use equations

$$\begin{cases} \dot{x}_3 = \begin{cases} -a_3x_3, \text{ if}(u = 0), \\ -k_3u, \text{ if}(u \neq 0), \end{cases} \\ u = \eta((x_3 - x_1), \delta), \end{cases} \quad (5)$$

where the signature function is implemented by the function $\eta(\ldots)$ of the relay element with hysteresis zone δ.

About the existence of sliding mode

Assertion. For a closed system (1) and (4) the existence of sliding trajectory in the form

$$s = a_s(t, x(t_0))x_1 + x_2 = 0 \quad (6)$$

is possible, where $a_s(t, x(t_0)) > 0$, $\forall t \in (t \geq t_0)$ if necessary conditions are met

$$\begin{gathered} x_1(t_0) = x_3(t_0), \\ a_3 \geq a_s(t). \end{gathered} \quad (7)$$

The Proof. Suppose that $\delta > \sigma$, where σ is an infinitely small positive number, not equal to zero. Let us introduce a function

$$q(t) = \frac{T_{(u\neq 0)}}{T_{(u\neq 0)} + T_{(u=0)}}$$

which is a duty factor of active control at given time intervals corresponding to one period of self-oscillation caused by the availability of hysteresis with σ value. The limit of this function converges to the finite number r

$$\lim_{\delta\to\sigma} q(t) \to r,$$

which takes values in the range [0, 1] and can be regarded as equivalent control [9]. Then the closed system model for $x_1(t_0) > 0$ can be written in the form of differential equations:

$$\begin{cases} \dot{x}_1 = x_2 = -a_s(t)x_1, \\ \dot{x}_2 = -hx_2 - gq(t), \\ \dot{x}_3 = -a_3(1 - q(t))x_1 + k_3q(t). \end{cases} \quad (8)$$

The differential equations (8) at given initial values of the phase variables $x_1(t_0) = C_1$, $x_2(t_0) = a_s(t_0)c_2$ have the only solution relative to the functions $a_s(t)x_1$, $q(t)$ relying on the assumption that the function $q(t)$ is equivalent control (4) providing fulfillment of the sliding mode condition $|x_1 - x_3| < \sigma$. Consequently if we have infinitesimal σ and equality $x_1 = x_3$ from the equation $a_s(t)x_1 = a_3(1 - q(t))x_1 - k_3q(t)$, we can obtain the condition:

$$q(t) = \frac{a_3 - a_s(t)}{a_3 + k_3x_1^{-1}} \leq 1,$$

where the solution relative to $x_1(t)$ has positive value along the whole sliding trajectory. Similarly, for the condition ($x_1(t_0) < 0$) the differential equations of the closed system can be written in the form:

$$\begin{cases} \dot{x}_1 = x_2 = -a_s(t)x_1, \\ \dot{x}_2 = -hx_2 + gq(t), \\ \dot{x}_3 = -a_3(1 - q(t))x_1 - k_3q(t). \end{cases} \quad (9)$$

The solution of the system of differential equations (9) allows us to write

$$q(t) = \frac{a_3 - a_s(t)}{a_3 - k_3x_1^{-1}} \leq 1,$$

where the state variable $x_1(t)$ has negative values along the whole sliding trajectory. Thus, for the variants under study, we can write the following equation:

$$q(t) = \frac{a_3 - a_s(t)}{a_3 + k_3|x_1^{-1}|} \leq 1.$$

Therefore the fulfillment of the condition (7) assumes positively defined values $q(t)$ in the range [0, 1]. Note that the choice of the value a_3 is determined by the conditions of the classical sliding mode existence [9], and the assertion

(7) testifies that the sliding mode (4) in the system (2) and (5) will be indirectly determined by the parameters of the object.

The conditions (6) and (7) are preceded by the initial phase of transient process in which the object trajectory is oriented towards the sliding trajectory [22]. The parameter k_3, which can be chosen so that condition (6) is not satisfied at the moment of the first exit to trajectory (7), has a significant effect on the nature of the exit to the sliding trajectory. Thus, the definition of the problem of this parameter selection can be done with the objective of time optimal control on base of the classical mode of sliding control (Fig. 1).

The properties of time optimal control

Let us consider the following statement which allows us to suggest available optimal properties of the relay control under study.

Assertion. For an object of type (1) accounting stationarity of the functions g, h and known initial state $\{x_1(t_0) = C_1 \in (C_{min}, C_{max}); x_2 = 0\}$, it is possible to organize time optimal control [17] corresponding to Pontryagin's principle of maximum in variations of the parameter k_3 during transient process.

The proof is based on the assertion about the number of switches of control effect for a sustained second-order object; in accordance with it optimal control with two different-polar impacts is provided. Let us show the solution of closed system of object control (1) and controller (2) in the form of transients in Fig. 2, *a*. According to the software-based negative feedback of the relay element the trajectory of the variable $x_3(t)$ (blue line) always trends to the trajectory of the variable x_1.

The rate of change of the variable x_3 is defined by the function $k_3(x, u)$ providing the intersection of the variables x_1 and x_3 at the point of switching of time optimal control. In particular, for a given initial state ($x_1 = 1$, $x_2 = 0$) of the control object, the function $k_3(x, u)$ equals to the absolute value of tangents of the angles between the phase trajectory x_3 and the time axis for two cases defined by the product sign of x_1 and $u - sign(x_1, u)$. Obviously the problem is solved for any initial position ($x_1 = x_1(t_0)$, $x_2 = 0$), and the algorithm to determine the position of control object phase state is very simple: it is a moment when the variable x_1 reaches maximum deviation from the equilibrium point.

By results of the analysis of the processes for time optimal control organization we propose a method to build the sliding trajectory parameter. Let us write the control (2) taking into account dependence of the configurable parameter k_3 on the initial state and sign

$$\begin{cases} u = \eta(x_3 - x_1, \delta), \\ \dot{x}_3 = \begin{cases} -a_3 x_3 & \text{if}(u = 0), \\ -u/T_{p1}(C_1) & \text{if}(sign(x_1 u) > 0), \\ -u/T_{p2}(C_1) & \text{if}(sign(x_1 u) > 0), \end{cases} \end{cases} \quad (10)$$

where the dependences $T_{p1}(C_1)$ and $T_{p2}(C_1)$ are determined for different initial states from the given set $C_1 \in (C_{min}, C_{max})$. The algorithm to determine dependences can be represented in several stages:

1) For a given control object there are defined boundary values $g = g_{min}$ and $h = h_{min}$ and for them optimal control is synthesized.
2) The value C_1 is calculated by solving the differential equations (1) at the reverse time at given duration of the second pulse of control signal switched by Fel'dbaum's method which does not require large computational resources. The example of such solutions is a family of transient processes $x_1(t)$, $x_3(t)$ for $C_1 \in (1,0; 0,8; 0,6; 0,4; 0,2)$ shown in Fig. 2, *b*.
3) On the base of the obtained transients we determine the values $T_{p1}(C_1)$ and $T_{p2}(C_1)$ for which there is a point of control switching.

The algorithm of implementation includes implementation of feedback and control in accordance with (10), the relay element and the algorithm of the control parameters recalculation in the beginning of transient process in accordance with Fig. 2, *b*. This implementation cannot be classified in the framework of existing conventional PID or linear–quadratic regulator (LQR) methods because there are program solutions along with differential solutions and complicated relay element in the algorithm. Also the considered algorithm cannot be defined as a controller with variable structure since the calculations are performed not in the phase plane. Note that the trajectories of the variables $x_1(t)$ and $x_3(t)$ on the time plane (Fig. 2, *a*) resemble the Greek letter `lambda` – λ. For this reason, within the framework of this article, the proposed method will be called the λ-regulator.

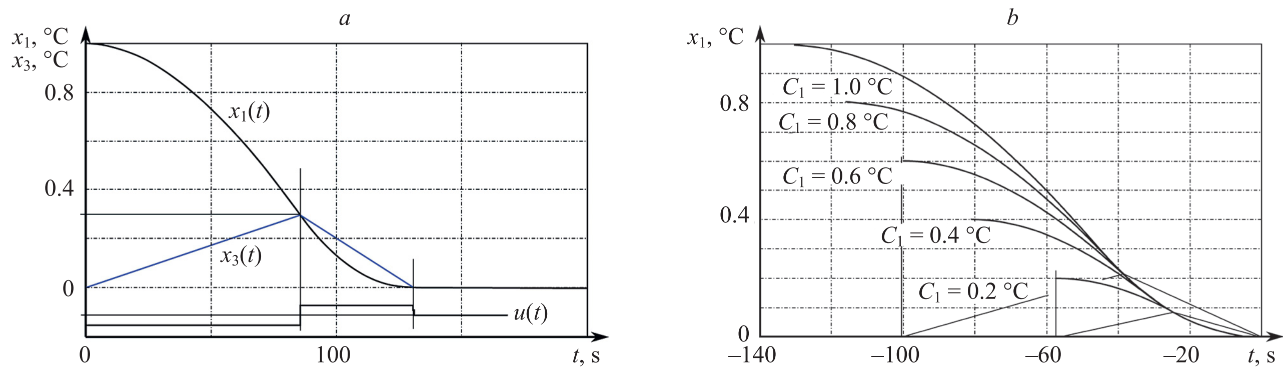

Fig. 2. Time optimal transient process (*a*); the family of transient processes (*b*)

Examination and comparison of λ-regulator as an alternative for PID-regulator

For comparison we select LQR-controller [19] where at the modeling stage we assume the following: all the state variables of the linearized system [23] are available for measurement, that's why no need to use the linear system state observer. Control values for LQR-regulator as opposed to relay control can include the whole set of values in the range [–1, +1]. In solution of Riccati equation, the weighting matrix **Q** is a single diagonal, and the weighting matrix **R** contains the only element whose value is chosen under the condition of control value limitation. The simulated results of the linearized system with LQR and λ-regulator are shown in Fig. 3.

Fig. 3, *a* shows transients of target variables T_{to3} and T_{bk1} and control u by solid lines for λ-regulator and by dotted lines for LQR-regulator.

The operating mode of a closed system is interesting to study when change of derivative from the target variable takes place because of number of reasons and external influences. Fig. 3, *b* demonstrates the simulated results allowing us to compare effectiveness of closed system operation. From the two figures we can see the advantages of time controller operation.

The operation of λ-regulator algorithm is compared with the operation of LQR-regulator which in consideration of the object astatism implements PI control law. For tradeoff with three-position control, pulse-width modulation is used. The result of the research of λ-regulator advantage can be observed in Fig. 4, *a* where two pulses of impact are clearly evinced in the beginning of the transient process.

To underscore the robust properties of the relay controller, two different heat-consumption objects were chosen: a large dwelling house and a garage box. The results of the research demonstrate that under change of operating time of drive stroke from 18 to 180 s and time of object inertia from 12 to 120 s the controller with fixed settings provides for both objects sliding mode with the trajectory time constant equal approximately 300 s (Fig. 4, *b*). Available range of change of drive stroke operating time and time of the object inertia allows us to cover almost the whole set of possible parameters of heat-consumption objects.

Relying on statistics of display of the relay controller robust properties in more than 500 technically operable heat units of heat-consumption circuits, there were ascertained typical controller settings: T_{p1} = 30 s, T_{p2} = 20 s, T_{ob} = 300 s.

It is advisable to separately consider the modes of active change in the flow of hot water in the heating system which leads to a characteristic perturbation of the observed temperature. Fig. 5 shows transients for these modes and almost even initial conditions of the heating system functioning with two types of λ-regulator and PID-regulator.

The comparative evaluation of the system target variable fluctuations allows us to conclude the advantage of the relay λ-regulator.

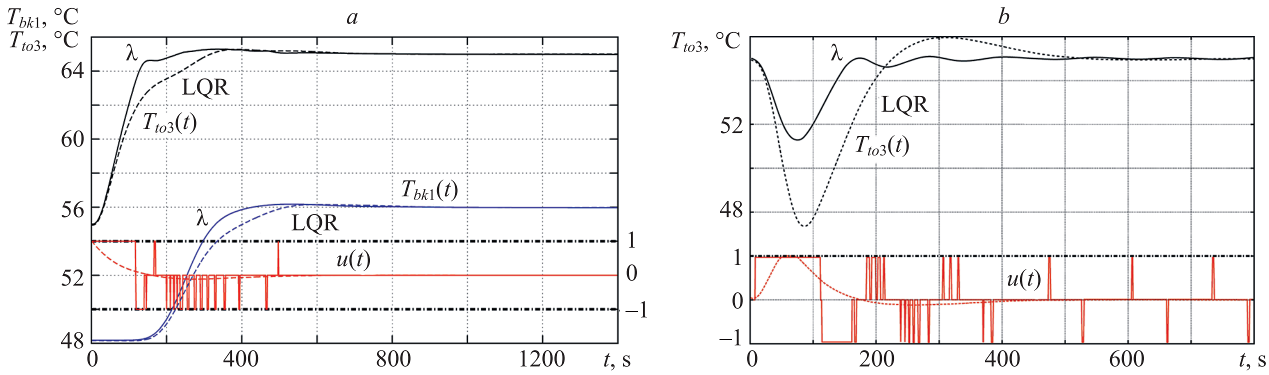

Fig. 3. Simulated results of transients of heat-consumption system with LQR and λ-regulators (*a*); the simulated result of operation under the change of the target variable derivative (*b*)

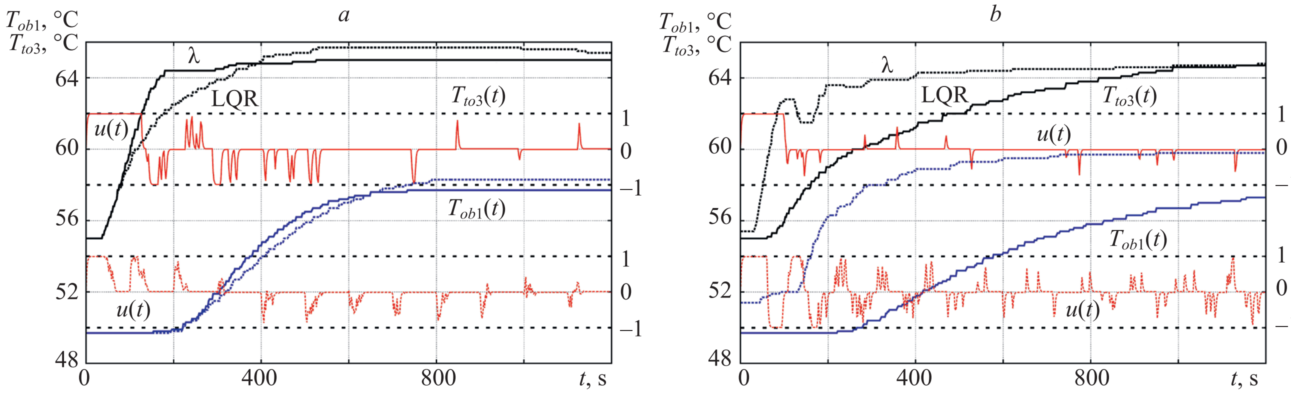

Fig. 4. Optimal control with relay and PI controllers (*a*); the robust properties of the relay λ-regulators (*b*)

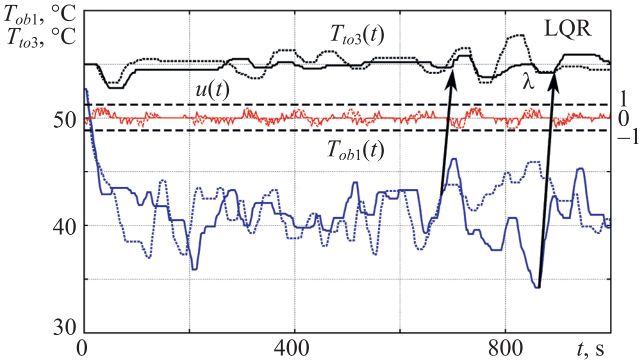

Fig. 5. Transients in active phase

The proposed λ-controller is a good alternative for replacing the PID controller with PWM in inertial-astatic systems, such as control of ventilation and heat consumption systems [24, 25]. This fact is confirmed by the successful operation of the λ-controller in the FBD of PLC units during 8 years.

Conclusion

The conducted studies show that the λ-regulator has an advantage in speed when controlling dynamic objects, in particular, for a deterministic second-order object with astaticism. The synthesis of λ-regulator is based on the principles of implementation of sliding modes having properties of time optimal control. At the same time, this regulator has robust sliding mode control properties for closed systems with interval parameters.

The obtained controller algorithm, implemented on a PLC, has been successfully used for several years at more than 100 heat supply facilities with three-position control. Experts in the field of control and automation can easily use the proposed algorithm for objects presented in the form of a mathematical model (1), without the need to calculate the derivative of the measured signal.

It is important to note that the proposed method cannot be offered as an unambiguous alternative to sliding control since the method of constructing a λ-controller and its software implementation have some peculiarities.

1. In particular, the properties of robust control are theoretically not strictly defined and not proven, as was done in relation to sliding control.
2. The a_s parameter (6) depends on the mismatch amplitude and decreases as the equilibrium point is approached. Thus, the question of the possibility of asymptotic stability with a constant parameter a_s remains unresolved in the theoretical presentation.
3. The software implementation algorithm is somewhat more complicated than simply calculating the sign of the control. It is obvious that the methods of adjusting the slip parameter to achieve the robust properties of the controller will lead to the complication of the algorithm.

A distinctive feature of the controller under consideration is the property of optimal control at high rates of change in processes in objects and with a large control error for objects represented by differential equations of the second and, possibly, third order.

The issue of the complexity in the control algorithm is not a significant obstacle for modern developed controllers. The presented drawbacks can be considered as open scientific tasks. These tasks can be useful and interesting to specialists in the field of mathematical analysis of nonlinear systems as an open challenge for further theoretical analysis.

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