

doi: 10.17586/2226-1494-2023-23-2-430-435

High performance modeling of the stress-strain state of thin-walled shell structures with the use of deep learning

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Abstract

Computer modeling is one of the most common approaches to the analysis of thin-walled shell structures stress-strain state analysis. It requires considerable time costs and high-performance hardware, especially when it is necessary to conduct a comparative analysis of various shell configurations. In this paper, we propose the use of deep learning methods to improve performance of this process. The purpose of work is to develop methods for high-performance computer simulation of thin-walled shell structures using deep neural networks, allowing to take into account geometric and physical properties of the structure as well as the load applied to it. A training approach and deep neural network architecture were developed to perform computer modeling of the stress-strain state of the shell. To form a training dataset, a computational experiment was carried out to simulate 3904 different configurations of doubly curved shallow shells that differ in linear dimensions, curvature radii, and materials used. Based on this dataset, 30 deep neural networks with different architectures were trained. To determine the optimal architecture in terms of modeling accuracy, mean absolute percentage error with clipping near-zero samples was calculated for each of the neural networks based on the test dataset. A network has been developed that allows calculating the stress-strain state of different shell configurations under an arbitrary uniformly distributed load. This is the first solution in the field of shell neural network modeling that allows us to vary the applied load, geometric and physical parameters of the shell and obtain calculation results at an arbitrary point of its middle surface. Performance measurements were carried out which show that the developed neural network allows simulating the stress-strain state of a shell structure 2117 times faster compared to the duration of solving the same problem by classical simulation. The modeling error using the network is at an acceptable level. An original architecture of a neural network for modeling the stress-strain state of shells was proposed which, through minor modifications, can be adapted for high-performance modeling of other building structure types. In accordance with the described architecture, a deep neural network was trained which reduces the computation time by several orders of magnitude. The results obtained are of high practical importance for researchers in the field of thin-walled shells modeling since they allow us to significantly reduce the time costs associated with conducting computational experiments. One of the possible applications for developed solution is prototyping of various shell configurations. Once prototyping is complete, the most efficient shell configurations can be explored in detail using classical computer simulation techniques.

Keywords

thin-walled shell structures, stress-strain state, deep learning, neural networks, computer modeling, Julia programming language

For citation: Zgoda Iu.N. High performance modeling of the stress-strain state of thin-walled shell structures with the use of deep learning. *Scientific and Technical Journal of Information Technologies, Mechanics and Optics*, 2023, vol. 23, no. 2, pp. 430–435. doi: 10.17586/2226-1494-2023-23-2-430-435

УДК 004.942+004.032.26

Высокопроизводительное моделирование напряженно-деформированного состояния тонкостенных оболочечных конструкций с использованием глубокого обучения

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Аннотация

Предмет исследования. Одним из наиболее распространенных подходов к исследованию напряженно-деформированного состояния тонкостенных оболочечных конструкций под воздействием внешних сил является их компьютерное моделирование. Данное решение требует существенных временных затрат и высокопроизводительного аппаратного обеспечения, особенно при необходимости проведения сравнительного анализа различных конфигураций оболочек. В данной работе для повышения производительности моделирования предложено применение методов глубокого обучения. Выполнена разработка высокопроизводительного метода компьютерного моделирования тонкостенных оболочечных конструкций с использованием глубоких нейронных сетей, позволяющего учесть геометрические и физические свойства конструкции, а также прикладываемую к ней нагрузку. **Метод.** Основа метода состоит в подходе к обучению и архитектуре глубокой нейронной сети, способной выполнять компьютерное моделирование напряженно-деформированного состояния оболочки. Для формирования обучающего набора данных проведен вычислительный эксперимент моделирования 3904 конфигураций пологих двояковыпуклых оболочек разных линейных размеров, радиусов кривизны и используемых материалов. Выполнено обучение 30 глубоких нейронных сетей различных архитектур. Для выбора архитектуры, оптимальной с точки зрения точности моделирования, для каждой из обученных сетей на проверочном наборе данных рассчитана средняя абсолютная ошибка в процентах с отсечением околонулевых образцов. **Основные результаты.** Разработана нейронная сеть, позволяющая без существенных вычислительных затрат определить напряженно-деформированное состояние множества конфигураций оболочек под воздействием произвольной равномерно-распределенной нагрузки. Данное решение — первое в области нейросетевого моделирования оболочек, позволяющее задавать прикладываемую нагрузку, геометрические и физические параметры оболочки и получать результаты расчета в произвольной точке срединной поверхности оболочки. Проведено сравнение производительности классического моделирования и моделирования напряженно-деформированного состояния разработанной нейронной сети. Для одной конструкции моделирование в нейронной сети выполняется в течение 2 мс, что в 2117 раз быстрее по сравнению с классическим. При этом погрешность моделирования с использованием сети получена на допустимом уровне. **Практическая значимость.** Предложена оригинальная архитектура нейронной сети моделирования напряженно-деформированного состояния пологих двояковыпуклых оболочек. Архитектура путем незначительных модификаций может быть приспособлена для высокопроизводительного моделирования разных видов строительных конструкций. Осуществлено обучение глубокой нейронной сети, которая обеспечивает сокращение длительности вычислений на несколько порядков. Полученные результаты обладают высокой практической значимостью для исследователей в области моделирования тонкостенных оболочечных конструкций. Наиболее перспективным применением разработанного решения является прототипирование различных конфигураций оболочек. По окончании прототипирования наиболее эффективные конфигурации могут быть детально исследованы с использованием классических методов компьютерного моделирования.

Ключевые слова

тонкостенные оболочечные конструкции, напряженно-деформированное состояние, глубокое обучение, нейронные сети, компьютерное моделирование, язык программирования Julia

Ссылка для цитирования: Згода Ю.Н. Высокопроизводительное моделирование напряженно-деформированного состояния тонкостенных оболочечных конструкций с использованием глубокого обучения // Научно-технический вестник информационных технологий, механики и оптики. 2023. Т. 23, № 2. С. 430–435 (на англ. яз.). doi: 10.17586/2226-1494-2023-23-2-430-435

Introduction

Thin-walled shell structures (or shells) are structures bounded by two curved surfaces, the largest distance between which is much smaller than any other dimension [1]. They are actively used in architecture and construction [2], aerospace [3], ship modeling [4], acoustics [5], space industry [6], and many other areas due to their low weight and high strength [7]. A visualization of a doubly curved shallow shell, which is one of the most common types of shell structures, is given below (Fig. 1).

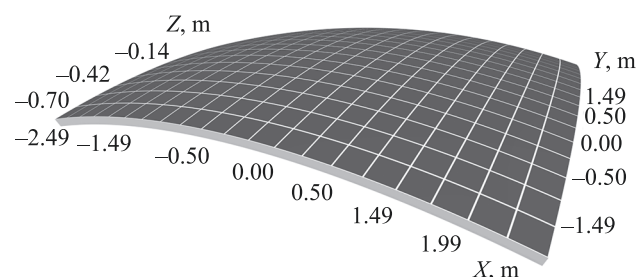


Fig. 1. Example of a doubly curved shallow shell

Computer modeling of a shell is time-consuming and requires high-performance hardware, such as multicore processors and graphical processors [8]. To analyze the properties of various shell configurations (which differ in their shapes and materials) it is necessary to calculate Stress-Strain State (SSS) of the shell that occurs under the influence of external forces. In accordance with the Reissner-Mindlin model [1] used in this work, shell SSS is described by a set of 5 functions of two spatial coordinates x, y : U, V, W (displacements in three orthogonal directions Ox, Oy, Oz accordingly) and Ψ_x, Ψ_y (functions of normal segment turning angles to the middle surface in planes xOz and yOz accordingly).

Suppose there are functions $\bar{U}, \bar{V}, \bar{W}, \bar{\Psi}_x, \bar{\Psi}_y$ which depend not only on x, y but also on geometry parameters and material properties as well as on the load applied to the shell. These functions describe the SSS of a whole class of shells. The presence of such functions would significantly reduce the time spent on the calculation. Obtaining an analytical expression for them seems impossible due to the complexity of the corresponding boundary value problem, so this work considers construction of their approximations with the use of deep learning.

The purpose of this work is to develop high-performance methods for computer simulation of thin-walled shell structures using deep neural networks and allowing to take into account geometric and physical properties of the structure as well as the load applied to it. To achieve this purpose, the following problems were solved:

- A computational experiment was carried out for 3904 shell configurations differing in geometric properties and material parameters.
- Architecture of an Artificial Neural Network (ANN) for modeling SSS of the shells is proposed.
- ANN was trained on the results of a computational experiment.
- Accuracy and performance of SSS modeling with the use of ANN was estimated.

During the work, a computer based on AMD Ryzen 9 3900X CPU, 64 GB RAM, Nvidia Geforce RTX 2070 Super GPU and OS Manjaro Linux (kernel version 5.15.44) was used. Julia programming language [9] was used for software development.

Application of deep learning in shell modeling is a relatively new scientific direction [10]. Therefore, the literature review also includes works on modeling plates (since a plate can be considered as special case of a shell without geometric curvilinearity). Mallela et al. [11] used the results of finite element modeling of composite plates to train a neural network capable of determining the critical load of the plate. The average error in determining critical load for that study was about 2 %. A similar approach was used in [12] and it has shown high effectiveness in calculating the critical load of hat-stiffened panels. Tahir et al. [13] used the data from a large amount of physical experiments on loading a cylindrical shell to train a critical load calculation ANN. They were able to achieve prediction accuracy that exceeds the accuracy of corresponding empirical formulas. These papers show high efficiency of ANNs in the problems of modeling plates and

shells. At the same time, reviewed network architectures allow determining only the critical load for a plate or a shell, while the proposed approach allows predicting displacements and rotation angles for a specific load at an arbitrary point of shell.

Ribeiro et al. [14] developed 4 ANNs SSS modeling for the plate (2 ANNs for modeling plates without reinforcement and 2 ANNs for modeling plates reinforced with ribs). The modeling accuracy of these networks was estimated by the authors as high. But in comparison to the proposed method, described ANNs do not allow varying thickness, physical characteristics of the plate, or the applied load. Also, the network output is limited to vertical displacements and von Mises stress value.

It is worth noting that ANNs trained on the results of physical experiments in most cases allow to achieve high modeling accuracy, and ANNs trained on the results of computer simulations are usually developed to increase modeling performance. When learning on “synthetic” dataset, ANN is limited by the accuracy of this dataset. From the other side, it usually requires much less computations for an ANN to transform the input data to SSS information.

Thereby, literature review has shown that the proposed approach (development of a neural network capable of determining SSS of the shell at any point of its surface by the value of applied load, geometric and physical properties) is an original solution of the discussed problem.

Training dataset preparation

To train an ANN predicting the SSS of a custom shell configuration, it is necessary to form a dataset describing SSSs of various shells. Therefore, it is necessary to prepare a set of pairs in the form of $(q, x, y, g_1, \dots, g_{N_g}, m_1, \dots, m_{N_m}) \Rightarrow (U, V, W, \Psi_x, \Psi_y)$, where q is the value of uniformly distributed load applied to the shell; x, y are coordinates of the considered point on the middle surface of the shell; g_1, \dots, g_{N_g} are the parameters describing shell geometry; m_1, \dots, m_{N_m} are the parameters describing shell material; N_g, N_m are the amounts of geometry parameters and material parameters, respectively.

We consider the class of doubly curved shallow shells. Geometry of these shells is determined by parameters a, b — linear dimensions along the Ox, Oy axes accordingly; h — shell thickness; and R_1, R_2 — curvature radii along Ox, Oy axes accordingly. Computer modeling was carried out with the assumption of material isotropy, therefore physical properties of the shell are determined by three parameters: E — the modulus of elasticity, G — the shear modulus, and μ — the Poisson's ratio.

When forming the training data set, doubly curved shallow shells with the following geometric characteristics were considered. The range of values for the linear dimensions is from 1.0 to 21.0 m and the step is 4 m. Range of thickness values is from 0.01 to 0.1 m and the step is 0.03 m. Range of values for the curvature radii is $[\min(a, b), 6 \min(a, b)]$ divided evenly by 3 parts (i.e., containing 4 evenly distributed values). From the described set of possible configurations, those configurations were excluded

for which $h > \frac{\min(a, b)}{20}$ (i.e., which are not thin walled).

Aluminum ($E = 7.00 \times 10^4$ MPa, $G = 2.60 \times 10^4$ MPa, $\mu = 0.34$) and steel ($E = 2.00 \times 10^5$ MPa, $G = 8.20 \times 10^4$ MPa, $\mu = 0.25$) were considered as construction materials. The applied load is uniformly distributed and normal to middle surface. The range of values for the load is from 0.0 MPa to 10.0 MPa in increments of 0.1 MPa. Shell termination is hinge-fixed.

In accordance with the procedure described above, 3904 configurations were formed. They were randomly divided into 80 % training configurations used for training dataset generation and 20 % test configurations used for test dataset generation. Then, for each configuration $101 = \frac{10.0}{0.1} + 1$ SSS calculations were performed in accordance with the previously defined load application interval. Total duration of computer modeling was around 7.9 hours. Overall amount of unique SSS calculations is 394,304 states.

Computer modeling of shells was carried out using author's software a detailed description of which is beyond the scope of this work. We only note that it is based on the application of Ritz method to solve the variation problem for total deformation energy functional. The Reissner-Mindlin model is used as a mathematical model when constructing the functional for a shell, and the LBFGS algorithm is used for minimization purposes. As a result of computer simulation, an approximate solution is formed as 5 functions $U(x, y)$, $V(x, y)$, $W(x, y)$, $\Psi_x(x, y)$, $\Psi_y(x, y)$ which are weighted sums of products of trigonometric functions.

To discretize continuous functions when preparing the training dataset, the surface of each shell was divided by a uniform grid of size 5×5 . This size was chosen because usually load-deflection diagrams are built in geometric center of the shell and its quarters while nodes of the 5×5 grid include these points. It is worth noting that using the same coordinates during each epoch of ANN training can lead to the state of neural network where its prediction for an arbitrary point (x, y) not falling in the center or one of shell quarters will be unreliable. Therefore, another training dataset was added which includes data from 5×5 grid with a non-uniform step determined by a pseudo-random number generator. This dataset was rebuilt every 5 epochs so that the network does not "get used" to the same coordinates. A similar approach was used to train ANN on shell behavior at its boundaries. Every 5 epochs, an additional training dataset was randomly generated from 20 points along the perimeter of the shell which guarantees correct training of the network to the conditions of shell termination.

In total, 27,601,280 pairs of the form $(q, x, y, a, b, h, R_1, R_2, E, G, \mu) \Rightarrow (U, V, W, \Psi_x, \Psi_y)$, i.e., pairs of input and output data, were generated. To optimize the ANN learning process, normalization was also applied to input data.

ANN architecture

Flux.jl library [15] was used to implement the neural network. To determine optimal configuration of the ANN, 30 network configurations were considered differing in architectures, amount of hidden layers, and sizes of each

layer. Mean Squared Error was used as a loss function and Mean Absolute Percentage Error (MAPE) with a cut-off for those samples that are less in modulus than $\varepsilon = 0.01$ was used as a metric for evaluating model quality. As a result of comparing different ANN configurations, the following was found to be the most effective:

- An input layer with 11 neurons by the number of parameters describing a shell.
- 4 dense layers with 176 neurons each (11×16).
- 5 parallel chains of layers, each with 1 layer of 187 neurons (skip connection from the input layer and 176 inputs from previous layer), 3 layers of 88 neurons (8 times the input length) and 1 layer with 1 output neuron.

Overall amount of trained parameters is equal to 327,365. ReLU activation function is used for all neurons except for the output layer; an identity function is used for the output layer. ADAM optimization procedure was applied for ANN training.

Approbation of ANN

ANN was trained for 50 epochs after which the values of loss function and MAPE metric on the test dataset stabilized at the level of 0.001 and 25 %, respectively. Considering the error of Reissner-Mindlin model, which is about 5–20 % depending on the degree of shell curvilinearity, achieved relative error is acceptable. Detailed analysis showed that the reason of high relative error is the incorrect prediction of near-zero values which is not critical for modeling purposes.

To demonstrate the capabilities of developed ANN, it was tested on a shell with the following characteristics: $a = 5.4$ m, $b = 5.4$ m, $h = 0.09$ m, $R_1 = 20.5$ m, $R_2 = 20.5$ m, $E = 2.0 \times 10^5$ MPa, $G = 8.2 \times 10^4$ MPa, $\mu = 0.35$. They were chosen to exclude the possibility of accurate ANN prediction due to overfitting the training data set. The only parameters that the network has already "seen" are the material parameters. Load-deflection diagrams obtained using the classical approach to calculation and ANN modeling are presented below (Fig. 2), where $W(q, x, y)$ is the vertical displacement of shell middle surface under load q at point (x, y) .

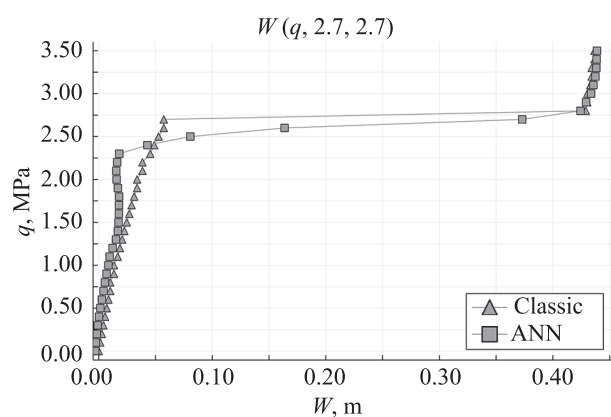


Fig. 2. Load-deflection diagram calculated by ANN and classic modeling methods

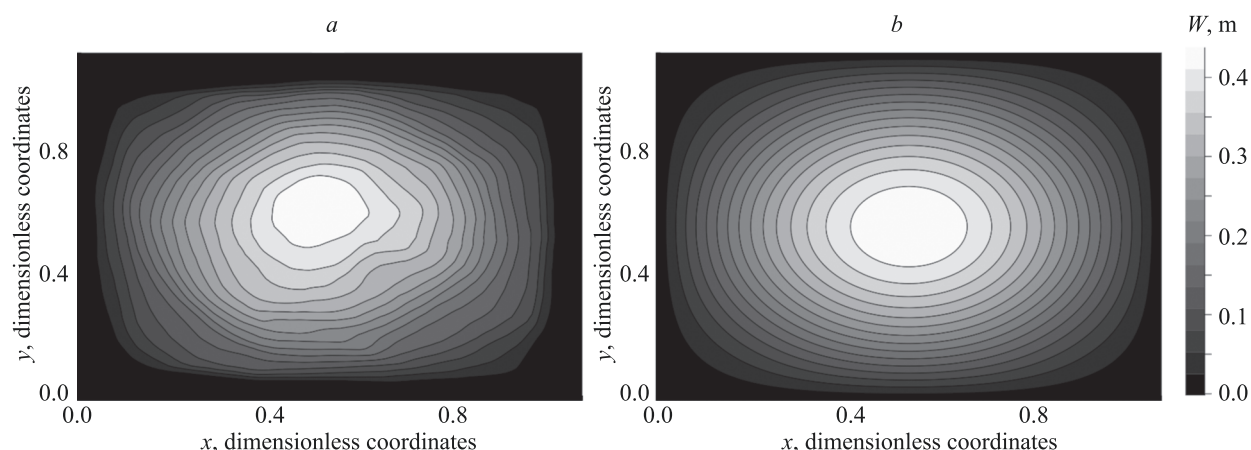


Fig. 3. Vertical displacement (W , m) of shell middle surface modeled by ANN (a) and classic modeling methods (b)

As can be seen from the figure, ANN provides high modeling precision. Displacement after the critical load ($q \approx 2.7$ MPa) was evaluated with relative error equal to 1.17 %. It is worth noting that ANN was able to determine the critical load of shell, but it could not simulate the “discreteness” of the transition to another stress-strain state after the critical load. Results of classic calculation with a load step of 0.1 MPa contains an abrupt change in the behavior of the structure when passing the 2.7 MPa load, but ANN performs the transition from one state to another relatively smoothly, starting with a load of 2.3 MPa and ending with a load of 2.8 MPa. On the other hand, all the necessary calculations took 0.002 s which is 2117 times faster compared to the classical computation approach, and this required 4.234 seconds for simulation.

The most efficient application of the proposed approach is modeling a large number of constructions. For example, to estimate the influence of shell dimensions on the critical load, researcher needs to model different configurations of shells (suppose 50 distinct values). Classical computer modeling approach will require about 3 hours (4.2 seconds per shell, $50 \times 50 = 2500$ computations). Using the proposed ANN, calculations will take about 5 seconds.

Contour maps of vertical displacement field $W(x, y)$ for the previously considered shell under load $q = 2.9$ MPa (i.e., after shell buckling) are presented below (Fig. 3). Coordinates x, y are dimensionless and obtained by normalizing the middle surface coordinates to the range

[0.0, 1.0], vertical displacements are given in meters. It can be seen that ANN (Fig. 3, a) has learned to correctly model shell fixing and that it describes shell behavior after critical load with high accuracy. Maximum absolute ANN modeling error is less than 1 cm.

To further evaluate ANN modeling accuracy, computer simulation was performed for 75 steel shells with the following characteristics: $a, b \in [7, 11, 15, 19, 23]$, $R_1 = R_2 = \min(a, b)$. These configurations do not intersect with training or test datasets. MAPE metric for them was equal to 36 % which is only 11 % higher than the metric value on the test dataset. In most cases large error values were caused by smooth transitions near the critical load value which is insignificant in most cases.

Conclusion

A deep neural network for stress-strain state computation of the shell was described and developed. Such a network makes it possible to increase the performance of shell modeling by several orders of magnitude. The proposed solution is unique and currently has no analogues. The most promising application of the developed artificial neural network is prototyping. It can be used for quick comparison of numerous structures. Most efficient configurations can be further examined with the use of classic computation methods. Main directions for further research are accuracy improvement and extension of the class of simulated structures.

References

1. Karpov V.V. Models of the shells having ribs, reinforcement plates and cutouts. *International Journal of Solids and Structures*, 2018, vol. 146, pp. 117–135. <https://doi.org/10.1016/j.ijsolstr.2018.03.024>
2. Hu N., Feng P., Dai G.-L. Structural art: Past, present and future. *Engineering Structures*, 2014, vol. 79, pp. 407–416. <https://doi.org/10.1016/j.engstruct.2014.08.040>
3. Chai Y., Song Z., Li F. Investigations on the aerothermoelastic properties of composite laminated cylindrical shells with elastic boundaries in supersonic airflow based on the Rayleigh–Ritz method. *Aerospace Science and Technology*, 2018, vol. 82–83, pp. 534–544. <https://doi.org/10.1016/j.ast.2018.09.040>
4. Liu H.-T., Li N. Reliability analysis of autonomous underwater vehicle aft pressure shell for optimal design and strength. *Ocean Engineering*, 2022, vol. 249, pp. 110906. <https://doi.org/10.1016/j.oceaneng.2022.110906>

Литература

1. Karpov V.V. Models of the shells having ribs, reinforcement plates and cutouts // *International Journal of Solids and Structures*. 2018. V. 146. P. 117–135. <https://doi.org/10.1016/j.ijsolstr.2018.03.024>
2. Hu N., Feng P., Dai G.-L. Structural art: Past, present and future // *Engineering Structures*. 2014. V. 79. P. 407–416. <https://doi.org/10.1016/j.engstruct.2014.08.040>
3. Chai Y., Song Z., Li F. Investigations on the aerothermoelastic properties of composite laminated cylindrical shells with elastic boundaries in supersonic airflow based on the Rayleigh–Ritz method // *Aerospace Science and Technology*. 2018. V. 82–83. P. 534–544. <https://doi.org/10.1016/j.ast.2018.09.040>
4. Liu H.-T., Li N. Reliability analysis of autonomous underwater vehicle aft pressure shell for optimal design and strength // *Ocean Engineering*. 2022. V. 249. P. 110906. <https://doi.org/10.1016/j.oceaneng.2022.110906>

5. Bevilacqua A., Ciaburro G., Iannace G., Lombardi I., Trematerra A. Acoustic design of a new shell to be placed in the Roman amphitheater located in Santa Maria Capua Vetere. *Applied Acoustics*, 2022, vol. 187, pp. 108524. <https://doi.org/10.1016/j.apacoust.2021.108524>
6. Lopatin A.V., Morozov E.V., Shatov A.V. Axial deformability of the composite lattice cylindrical shell under compressive loading: Application to a load-carrying spacecraft tubular body. *Composite Structures*, 2016, vol. 146, pp. 201–206. <https://doi.org/10.1016/j.compstruct.2016.03.021>
7. Zhang Y., Song H., Yu X., Yang J. Modeling and analysis of forced vibration of the thin-walled cylindrical shell with arbitrary multi-ring hard coating under elastic constraint. *Thin-Walled Structures*, 2022, vol. 173, pp. 109037. <https://doi.org/10.1016/j.tws.2022.109037>
8. Kostopanagiotis C., Kopanos M., Ioakim D., Perros K., Lagaros N.D. Low cost CPU–GPGPU parallel computing in real-world structural engineering. *Journal of Building Engineering*, 2015, vol. 4, pp. 209–222. <https://doi.org/10.1016/j.jobbe.2015.09.011>
9. Bezanson J., Edelman A., Karpinski S., Shah V.B. Julia: a fresh approach to numerical computing. *SIAM Review*, 2017, vol. 59, no. 1, pp. 65–98. <https://doi.org/10.1137/141000671>
10. Thai H.T. Machine learning for structural engineering: A state-of-the-art review. *Structures*, 2022, vol. 38, pp. 448–491. <https://doi.org/10.1016/j.istruc.2022.02.003>
11. Mallela U.K., Upadhyay A. Buckling load prediction of laminated composite stiffened panels subjected to in-plane shear using artificial neural networks. *Thin-Walled Structures*, 2016, vol. 102, pp. 158–164. <https://doi.org/10.1016/j.tws.2016.01.025>
12. Sun Z., Lei Z., Bai R., Jiang H., Zou J., Ma Y., Yan C. Prediction of compression buckling load and buckling mode of hat-stiffened panels using artificial neural network. *Engineering Structures*, 2021, vol. 242, pp. 112275. <https://doi.org/10.1016/j.engstruct.2021.112275>
13. Tahir Z.R., Mandal P., Adil M.T., Naz F. Application of artificial neural network to predict buckling load of thin cylindrical shells under axial compression. *Engineering Structures*, 2021, vol. 248, pp. 113221. <https://doi.org/10.1016/j.engstruct.2021.113221>
14. Ribeiro J.P., Tavares S.M., Parente M. Stress-strain evaluation of structural parts using artificial neural networks. *Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications*, 2021, vol. 235, no. 16, pp. 1271–1286. <https://doi.org/10.1177/1464420721992445>
15. Innes M., Saba E., Fischer K., Gandhi D., Rudilosso M.C., Joy N.M., Karmali T., Pal A., Shah V. Fashionable modelling with Flux. *arXiv*, 2018, arXiv.1811.01457. <https://doi.org/10.48550/arXiv.1811.01457>

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Received 09.07.2022

Approved after reviewing 10.02.2023

Accepted 22.03.2023

Статья поступила в редакцию 09.07.2022

Одобрена после рецензирования 10.02.2023

Принята к печати 22.03.2023



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