

АВТОМАТИЧЕСКОЕ УПРАВЛЕНИЕ И РОБОТОТЕХНИКА AUTOMATIC CONTROL AND ROBOTICS

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Frequencies estimation in multisinusoidal time-varying parameter of the first order discrete linear system with the application to indirect adaptive control

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Abstract

The paper addresses the problem of adaptive frequencies estimation for multisinusoidal Time-Varying (TV) parameter of a discrete linear system of the first order. It is assumed that the amplitudes, frequencies, and phases of the harmonics in the TV parameter are unknown, however the number of harmonics is known. The novelty of the proposed approach consists in the fact that the frequencies identification is possible even if the system output crosses zero when the information about the TV parameter is inaccessible. In this case, when the proposed solution is used in a problem of adaptive control of the system considered, the frequencies identification and the work of TV parameter observer are independent, what increases the rate and precision of controller parameters tuning. The problem is solved by transformation of the plant model into a regression model linear with respect to unknown frequencies and used for design of identification algorithms. In the paper, two identification algorithms are applied. The first one is the standard gradient algorithm, while the second one is the algorithm with improved parametric convergence achieved by accumulation of regressor over past period of time and referred to as algorithm with memory regressor extension. The problem of control is solved with the use of: certainty equivalence principle; internal model principle according to which the TV parameter is represented as the output of dynamic autonomous model (exosystem) and involving of this model into the structure of the control law; observer of the exosystem state; one of the proposed frequencies identifier; and a formula of recalculation of the frequencies estimates into the controller adjustable parameters. A procedure of transformation of the TV system into a regression linear with respect to unknown frequencies used for design of identification algorithms is represented. The obtained solution is applied to the problem of indirect (identification-based) adaptive control of the TV system considered in the paper. The main distinguishing feature of the solution proposed consists in independence of the obtained identifiers from the observation property of the TV parameter what increases the transient performance and precision of the indirect adaptive control algorithms designed for the considered class of TV systems. The proposed solution can be used in problems of control of parametric resonance systems.

Keywords

linear time-varying system; multisinusoidal time-varying parameter; uncertain periodic coefficients; adaptive identification; adaptation with improved convergence

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Оценивание частот мультисинусоидального переменного параметра дискретной линейной системы первого порядка с применением в задаче непрямого адаптивного управления

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Аннотация

Введение. Рассмотрена задача адаптивного оценивания частот мультисинусоидального переменного во времени параметра дискретной линейной системы первого порядка с применением результата в задаче адаптивного управления. Предполагается, что амплитуды, частоты и фазы гармоник переменного параметра неизвестны, однако известно число этих гармоник. Новизна предлагаемого подхода заключается в том, что идентификация частот происходит даже при пересечении выходом системы нуля, когда недоступна информация о нестационарном параметре. В этом случае при использовании предложенного решения в задаче адаптивного управления рассматриваемым объектом идентификация частот и работа наблюдателя динамики переменного параметра происходят независимо, что повышает скорость и точность автонастройки параметров регулятора. **Метод.** Задача решается путем преобразования модели объекта к линейной по неизвестным частотам регрессии, на базе которой синтезируются алгоритмы идентификации. В работе применяется два алгоритма идентификации. Первый представляет собой стандартный градиентный алгоритм, в то время как второй — алгоритм адаптации с улучшенной сходимостью, обеспеченной за счет накопления предыдущих значений регрессора, и называется алгоритмом адаптации с памятью регрессора. Задача управления решена с использованием: принципа непосредственной компенсации; принципа внутренней модели, согласно которому переменный параметр представляется в виде выхода динамической автономной модели (экзосистемы) и внедрении этой модели в структуру закона управления; наблюдателя состояния экзосистемы; одного из двух предложенных идентификаторов частоты; формулы пересчета оценок частот в вектор настраиваемых параметров регулятора. **Основные результаты.** Приведена процедура сведения нестационарной системы к линейной по неизвестным частотам регрессионной модели, на базе которой построены алгоритмы идентификации. Полученное решение использовано в задаче непрямого (идентификационного) адаптивного управления рассматриваемой нестационарной системы. **Обсуждение.** Главной отличительной чертой предложенного решения является независимость полученных алгоритмов идентификации от свойства наблюдаемости нестационарного параметра, что повышает быстродействие и точность алгоритмов непрямого адаптивного управления рассматриваемым классом линейных нестационарных систем. Представленное решение может быть использовано при решении задач управления системами с параметрическим резонансом.

Ключевые слова

линейная нестационарная система, мультисинусоидальный переменный параметр, неопределенные периодические коэффициенты, адаптивная идентификация, адаптация с улучшенной сходимостью

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Introduction

In this paper, we develop an adaptive estimation technique for a class of discrete linear Time-Varying (TV) systems with uncertain multi-sinusoidal parameters and without disturbance.

Attempts of extension of identifiers and controllers design to TV systems made for the last decades are very challenging; however there exist potentials for increasing the scope of practical applications. One of the ways of such an extension is based on the Internal Model Principle (IMP) [1, 2] consisting in representation of the system TV parameters as the outputs of autonomous models (referred to as exosystems) with constant parameters and suitable incorporation of the structure of these models into the problem solution to completely compensate the effects caused by the TV parameters changing. In this case, as

shown in [2], the exosystems can generate multisinusoidal functions of time, and, as a result, the corresponding design methods can be successfully used for systems with multisinusoidal TV parameters and then applied to different real-life systems [3–6].

From the practical point of view, assumption about the complete knowledge of the system TV parameters is hardly feasible, hence the interest of adaptive control and identification theory application. At the same time, as the literature survey showed, the overwhelming majority of identification based on direct adaptive controllers for systems with multisinusoidal or just periodic coefficients assume that the amplitudes and phases of the coefficients are unknown; however the periods or frequencies of the parameters variation are known [3, 7–12] or partially known (in [13] the frequency is factorized by an unknown integer and a known constant, in [10, 12] the frequency is unknown

within a known interval). In [14], the authors propose an estimation technique for a linear system with sinusoidal parameters with completely unknown amplitudes, phases, and frequencies, however under assumptions that the plant state is bounded away from zero and the TV parameters contain only one harmonic. In [15, 16], the authors propose an approach of direct adaptive control for similar class of continuous time and discrete time linear systems with the state matrix containing multisinusoidal TV parameters on the main diagonal. The main drawback of the proposed method consists in dependence of controller parameters estimation (adaptation) on the plant state. So, if at least one element of the state vector crosses zero, the TV parameter is no longer to be observable and, as a result, the adaptive estimator being dependent on the TV parameters observers is not able to estimate the controller parameters.

Thus, in this paper we make an effort to overcome this problem by conversion of plant model into a linear regression with the regressor that is independent from the observer estimates and admits the state zero crossing. The regression is used for design of two frequencies estimators. The first estimator is based on the standard gradient adaptation algorithm, while the second one uses modified Kreisselemeier-like adaptation algorithm referred to as adaptation algorithm improved parametric convergence achieved by Memory Regressor Extension (MRE) [2, 17, 18]. Both estimators are applied to the proposed scheme of indirect (identification based) adaptive control.

In order to focus the readers' attention on the main idea and simplify the paper presentation, we start considering a discrete TV linear system of the first order with one multisinusoidal parameter in which, besides the unknown frequencies of the harmonics, the amplitudes and phases of these harmonics are assumed unknown. Then, it is shown how the proposed method can be straightforwardly extended to the more general class of linear systems considered in [16].

The remaining of the paper is organized as follows. In the second section, the problem of the frequencies identification is formulated. In the third section, the parameterization of plant model is presented. In the fourth section, the adaptive frequencies estimators are designed. In the fifth section, the identification algorithms are applied to the scheme of indirect adaptive control.

For the sake of completeness of the paper presentation, we represent the definition of the Persistent Excitation (PE) condition crucial for systems identification.

Definition 1. A bounded function $\varphi(k) \in \mathbb{R}^N$ is persistently exciting (i.e., $\in \mathcal{PE}$) if there exist such constants $\alpha, K \in \mathbb{R}_+$ such that:

$$\sum_{j=k+1}^{k+K} \varphi(j) \varphi^T(j) \geq \alpha \mathbf{I}_n \quad \forall k \geq 0. \quad (1)$$

Notations: z^{-1} is the delay operator; k is the sampling time; $\mathbb{N}$ is the set of natural numbers; $\mathbb{R}_+$ is the set of positive real numbers; $\mathcal{PE}$ is the set of persistently exciting functions satisfying (1); $\mathbf{I}_n$ is the $n \times n$ identity matrix; $\mathcal{L}_2$ is the space of squared summable functions; $\mathcal{L}_\infty$ is the space of bounded functions; $C_N^i = \frac{N!}{i!(N-i)!}$ is the binomial coefficient.

Problem statement

Consider a discrete-time system

$$x(k+1) = \psi(k)x(k) + u(k), \quad (2)$$

where $x(k) \in \mathbb{R}$ is the state with the initial condition $x(0)$, $u(k) \in \mathbb{R}$ is the control signal, k is the sampling time, $\psi(k) \in \mathbb{R}$ is the uncertain time-varying parameter given by the function

$$\psi(k) = \sum_{i=1}^N a_i \sin(\omega_i kT + \varphi_i) \quad (3)$$

with a priori unknown constant amplitudes a_i , phases φ_i , and frequencies ω_i of harmonics, however known maximum number of harmonic N and the discrete time interval T^1 .

For the plant, we accept the following assumptions.

Assumption 1.

- A1.1. The frequencies ω_i are different (distinguishable) and satisfy the conditions $\omega_i T \in (0; \pi]$;
- A1.2. None of the frequencies ω_i is zero, i.e., the signal (3) is not biased.

Assumption A1.1 allows us to identify the frequencies and is typical for harmonic identification problems. Assumption A1.2 is introduced for the sake of simplicity and can straightforwardly be extended to biased (i.e., with zero frequency) multisinusoidal parameters.

The objective is to design an identifier that under the PE condition generates estimates $\hat{\omega}_i(k)$ such that

$$\lim_{k \rightarrow \infty} (\omega_i - \hat{\omega}_i(k)) = 0. \quad (4)$$

It is necessary to note that compared to the identifier (adaptation algorithm) proposed in [16], this problem statement admits that the state $x(k)$ can cross zero arbitrary number of times; therefore the solution presented below is less restrictive.

The solution proposed is based on linear parameterization of the plant model (2) superposition of measurable variables multiplied by constant parameters dependent from ω_i . Before to propose the general solution to the problem, we clarify the main idea of the parameterization considering the particular case of single harmonic parameter (3) with $N = 1$ and then extend this case to arbitrary $N > 1$.

Model parameterization

Single harmonic case

Assuming that

$$\psi(k) = a \sin(\omega kT + \varphi),$$

where a , ω , and φ are constant unknown parameters, and applying the IMP we represent $\psi(k)$ as the solution of the linear discrete equation

$$\beta(z)[\psi(k)] = 0 \quad (5)$$

with the operator $\beta(z) = 1 - cz^{-1} + z^{-2}$; the initial conditions $\psi(0) = a \sin(\varphi)$; $\psi(1) = a \sin(\omega T + \varphi)$ and the constant $c = 2\cos(\omega T)$.

¹ Hereafter, we will omit the dependence from k for the sake of brevity and if is not in contrary to the context.

From (2) we obtain

$$\psi(k) = \frac{x(k+1) - u(k)}{x(k)}. \quad (6)$$

Replacing $\psi(k)$ from the latter in (5), we have

$$\frac{x(k+1) - u(k)}{x(k)} - c \frac{x(k) - u(k-1)}{x(k-1)} + \frac{x(k-1) - u(k-2)}{x(k-2)} = 0.$$

Multiplying this equality by $x(k)x(k-1)x(k-2)$, applying the shift backward operator z^{-1} , and rearranging terms, we obtain the parameterized model of the plant in the form of linear regression

$$y(k) = \theta \varphi(k), \quad (7)$$

where $\theta = c = 2\cos(\omega T)$,

$$y(k) = x(k-2)x(k-3)(x(k) - u(k-1)) + x(k-1)x(k-2)(x(k-2) - u(k-3)),$$

$$\varphi(k) = x(k-1)x(k-3)(x(k-1) - u(k-2)).$$

Remark 1. The form (7) can be used for identification of the frequency estimate $\hat{\omega} = \frac{1}{T}a \cos\left(\frac{1}{2}\hat{\theta}\right)$, where $\hat{\theta}$ is the estimate of θ generated by an identifier, under Assumption 1.1 since the function $\cos(\cdot)$ is invertible in the interval $(0, \pi]$.

Now, extend the presented result to arbitrary number of harmonics.

Multiharmonic case

Now, assume that $\psi(k)$ is represented in the general form (3). In this case, the operator form of discrete equation generating $\psi(k)$ takes the form

$$\prod_{i=1}^N \beta_i(z) [\psi(k)] = 0 \quad (8)$$

with the operators $\beta_i(z) = 1 - c_i z^{-1} + z^{-2}$, the initial conditions $\psi(j) = \sum_{i=1}^N a_i \sin(\omega_j T + \varphi_i)$ ($j = 0, 1, \dots, 2N-1$), and the constants $c_i = 2\cos(\omega_i T)$.

Replacing $\psi(k)$ from (6) in (8) after straightforward but routine algebraic transformations we obtain the following result.

Lemma 1. The plant (2) with the parameter (3) can be transformed into the form of linear regression

$$y(k) = \theta^T \varphi(k), \quad (9)$$

where $\varphi = [\varphi_1, \varphi_2, \dots, \varphi_N]^T$ is the regressor with the elements

$$\varphi_i(k) = \sum_{j=0}^{N-i} C_j \left(\prod_{\substack{p=1 \\ p \neq 2j+i+1}}^{2N+1} x(k-p) \right) (x(k-2j-i) - u(k-2j-i-1)), \quad (10)$$

$$y(k) = \sum_{i=0}^N C_i \left(\prod_{\substack{j=1 \\ j \neq 2i+1}}^{2N+1} x(k-j) \right) (x(k-2i) - u(k-2i-1)), \quad (11)$$

is the regression output; $\theta = [\theta_1 \ \theta_2 \ \dots \ \theta_N]^T$ is the vector obtained using c_i via the Vieta's formulas

$$\begin{cases} \theta_1 = c_1 + c_2 + \dots + c_N, \\ \theta_2 = -(c_1 c_2 + c_1 c_3 + \dots + c_{N-1} c_N), \\ \vdots \\ \theta_N = (-1)^{N+1} c_1 c_2 \dots c_N. \end{cases} \quad (12)$$

The linear regression (9) can be used for identification of the parameters θ_i ($i = 1, 2, \dots, N$) and, as a result, of the frequencies ω_i .

Adaptive frequencies identification

In order to estimate θ in (9), we use the gradient adaptation algorithm [19, 20] and the adaptation algorithm with improved parametric convergence based on the Kreisselmeier's like scheme and referred to as algorithm with MRE [17, 18].

Gradient algorithm

The algorithm takes the form

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \gamma \frac{\varphi(k)\varphi^T(k)}{1 + \gamma \varphi^T(k)\varphi(k)} \varepsilon(k), \quad (13)$$

where $\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k)$, $\gamma > 0$ is adaptation gain.

The algorithm (13) yields the parametric error model

$$\tilde{\theta}(k+1) = \left(\mathbf{I}_N - \gamma \frac{\varphi(k)\varphi^T(k)}{1 + \gamma \varphi^T(k)\varphi(k)} \right) \tilde{\theta}(k), \quad (14)$$

where $\tilde{\theta}(k) = \theta - \hat{\theta}(k)$ is the parametric error vector. This model can be used for the proof of the following properties.

Proposition 1. The identification system closed by the algorithm (13) with φ defined in (10) and y defined by (11) has the following properties:

- P1.1. $\tilde{\theta}(k) \in \mathcal{L}_\infty \cap \mathcal{L}_2$. If $\varphi(k) \in \mathcal{L}_\infty$, then $\varepsilon(k)$, $\hat{\theta}(k+1) - \hat{\theta}(k) \in \mathcal{L}_\infty \cap \mathcal{L}_2$;
- P1.2. If $\varphi(k) \in \mathcal{L}_\infty$, then $\varepsilon(k)$, $\hat{\theta}(k+1) - \hat{\theta}(k) \rightarrow 0$ as $k \rightarrow \infty$;
- P1.3. $\tilde{\theta}(k) \rightarrow 0$ exponentially fast as $k \rightarrow \infty$ if and only if $\varphi(k) \in \mathcal{PE}$. As a result, the objective (4) is achieved;
- P1.4. If $\tilde{\theta}(k) \rightarrow 0$ as $k \rightarrow \infty$, then there exists an optimal value of γ for which the rate of convergence is maximum.

The proposition can be proved using the Lyapunov function

$$V(k) = \frac{1}{2\gamma} \tilde{\theta}^T(k) \tilde{\theta}(k) \quad (15)$$

and evaluating its shift forward value $V(k+1)$ in view of (14) [19].

Unfortunately, the algorithm suffers from the drawback defined in Property P1.4 according to which the maximum rate of parametric convergence can be arbitrarily small and unacceptable from the practical point of view. In order to overcome this problem, in the next subsection we represent the adaptation algorithm with MRE that under the PE condition dramatically increases the rate of parametric convergence [18].

Scheme with MRE

We introduce an asymptotically stable positive transfer function of the form

$$L(z) = \prod_{i=1}^M \frac{1-d_i}{z-d_i}, \quad (16)$$

where $0 < d_i < 1$ are the constant design parameters, $M \in \mathcal{N}$. Premultiplying (9) by $\boldsymbol{\varphi}(k)$ and then applying the operator $L(z)$ we get the memory extended regression model

$$\mathbf{Y}(k) = \boldsymbol{\Omega}(k)\boldsymbol{\theta} \quad (17)$$

with the memory extended output $\mathbf{Y}(k) = L(z)[\boldsymbol{\varphi}(k)y(k)]$ and the memory extended regressor matrix $\boldsymbol{\Omega}(k) = L(z)[\boldsymbol{\varphi}(k)\boldsymbol{\varphi}^T(k)]$.

The regression model (17) allows us to design the adaptation algorithm with MRE

$$\hat{\boldsymbol{\theta}}(k+1) = \hat{\boldsymbol{\theta}}(k) + \gamma(\mathbf{I}_N + \gamma\boldsymbol{\Omega}(k))^{-1}\mathbf{E}(k), \quad (18)$$

where $\mathbf{E}(k) = \mathbf{Y}(k) - \boldsymbol{\Omega}(k)\hat{\boldsymbol{\theta}}(k)$.

The algorithm (18) yields the parametric error model

$$\tilde{\boldsymbol{\theta}}(k+1) = (\mathbf{I}_N - \gamma(\mathbf{I}_N + \gamma\boldsymbol{\Omega}(k))^{-1}\boldsymbol{\Omega}(k))\tilde{\boldsymbol{\theta}}(k) \quad (19)$$

which can be used for the proof of the following properties.

Proposition 2. *In the closed-loop identification system, the algorithm (18) with $\boldsymbol{\varphi}$ defined in (10), y defined by (11), and $L(z)$ given by (16) provides Properties P1.1, P1.2, P1.3 and:*

— P2.1. *If $\tilde{\boldsymbol{\theta}}(k) \rightarrow 0$ as $k \rightarrow \infty$, then the rate of parametric convergence can be increased by increasing γ .*

The proposition is proved via the Lyapunov function (15) and its shift forward value $V(k+1)$ in view of (19) [18].

Frequencies estimation

Replacing in (12) θ_i and c_i ($i = 1, 2, \dots, N$) by their estimates $\hat{\theta}_i$ generated by the algorithm of adaptation (13) or (18) and the estimates $\hat{c}_i$, respectively, we obtain the system of algebraic equations

$$\begin{cases} \hat{\theta}_1 = \hat{c}_1 + \hat{c}_2 + \dots + \hat{c}_N, \\ \hat{\theta}_2 = -(\hat{c}_1\hat{c}_2 + \hat{c}_1\hat{c}_3 + \dots + \hat{c}_{N-1}\hat{c}_N), \\ \vdots \\ \hat{\theta}_N = (-1)^{N+1}\hat{c}_1\hat{c}_2 \dots \hat{c}_N. \end{cases} \quad (20)$$

Solving the latter, we get $\hat{c}_i$ and taking into account Assumption 1 calculate the frequencies $\hat{\omega}_i(k)$ via the following equalities:

$$\hat{\omega}_i(k) = \frac{1}{T} \arccos(\hat{c}_i(k)). \quad (21)$$

Now, discuss the obtained result according to the properties defined by Propositions 1 and 2.

Remark 2.

- R2.1. As shown in the next section, the boundedness of $\boldsymbol{\varphi}(k)$ can be provided via suitable design of input $u(k)$;
- R2.2. According to the properties of discrete-time systems, for the adaptation algorithm with MRE (18) the rate of parametric convergence can be increased up to N steps;

- R2.3. The PE property of $\boldsymbol{\varphi}(k)$ depends on the input signal $u(k)$. As discussed in the next section, the certainty equivalent control law provides the PE condition for $x(t) \not\equiv 0$ in the interval of excitation $[k+1, k+K]$ (see Definition 1) even if the reference signal is zero;
- R2.4. Transient solution of the system (20) always exists as a result of the Vieta's theorem, however can be complex. Therefore, in order to implement $\hat{\omega}_i(k)$ instead of (21), we use

$$\hat{\omega}_i(k) = \frac{1}{T} \arccos(\text{Real}\{\hat{c}_i(k)\}),$$

where $\text{Real}\{\cdot\}$ is the real part of a complex number.

In the next section, we demonstrate the application of the proposed identifier in the identification-based adaptive control of the plant (2).

Application: identification-based adaptive control

Problem statement

The problem is to design a control law that being applied to the plant (2) will ensure the boundedness of all the closed-loop signals in the system and:

O1. if $|x(k)| \geq x_0$ for all $k \geq N_0$ ($N_0 \in \mathcal{N}$), where $x_0 \in \mathbb{R}_+$ is a predefined threshold, then it will guarantee the limiting equality

$$\lim_{k \rightarrow \infty} \varepsilon(k) = \lim_{k \rightarrow \infty} (x_m(k) - x(k)) = 0 \quad (22)$$

in which $x_m(k)$ is a bounded reference signal with known next step value $x_m(k+1)$;

O2. otherwise it will guarantee the limiting inequality

$$\lim_{k \rightarrow \infty} |\varepsilon(k)| \leq \Delta, \quad (23)$$

where $\Delta \in \mathbb{R}_+$ is a maximum steady state error that is assumed to be reduced by changing design parameters.

Below, we represent the design of identification-based adaptive controller for the plant (2). The key distinguishable property of this controller compared to the existing one from [16] consists in separate design of adaptive identifier tolerant to the state zero crossing and an adjustable controller. The problem solution is calculated in three steps:

1. Design of an observer of the TV parameter $\psi(k)$ required to recover some information about the parameter dynamics;
2. Design of an adjustable control law;
3. Recalculation of the estimates $\hat{\boldsymbol{\theta}}$ given by the identifier obtained in the previous and its replacement in the control law designed.

TV parameter observer

In order to design a control law, it is needed to recover the information about the TV parameter dynamics. To this end, we, first, apply the IMP and represent the parameter $\psi(k)$ as the output of the exosystem given by the partial case of the canonical form recently presented in [16].

Lemma 2. The TV parameter can be represented as the output of the linear regression

$$\psi(k) = \boldsymbol{\vartheta}^T \boldsymbol{\xi}(k), \quad (24)$$

where $\xi(k) \in \mathbb{R}^{2N}$ is the state of the filter

$$\xi(k+1) = \mathbf{G}\xi(k) + \mathbf{I}\psi(k) \quad (25)$$

with the initial condition $\xi(0) = [\psi(0), \psi(1), \dots, \psi(2N-1)]$, the matrix

$$\mathbf{G} = \begin{bmatrix} \mathbf{O}_{(2N-1) \times 1} & \mathbf{I}_{(2N-1)} \\ 0 & \mathbf{O}_{1 \times (2N-1)} \end{bmatrix},$$

m -dimensional vector $\mathbf{I} = [0, 0, \dots, 0, 1]^T$ selected so that the pair $(\mathbf{G}, \mathbf{I})$ is controllable, and the vector of unknown parameters $\boldsymbol{\vartheta} \in \mathbb{R}^{2N}$ given by

$$\boldsymbol{\vartheta} = [\bar{\boldsymbol{\vartheta}}_1^T, \bar{\boldsymbol{\vartheta}}_2^T, \dots, \bar{\boldsymbol{\vartheta}}_N^T]^T \quad (26)$$

with

$$\bar{\boldsymbol{\vartheta}}_1 = \begin{bmatrix} -1 \\ \theta_1 \end{bmatrix},$$

$$\bar{\boldsymbol{\vartheta}}_j = \begin{cases} \begin{bmatrix} -C & + \sum_{i=1}^{j-1} C & \theta_{2i} \\ N & j-i & N-2i \\ \sum_{i=1}^j C & \theta_{2i-1} \end{bmatrix} & \text{if } 2j \leq N+1, \\ \begin{bmatrix} -C & + \sum_{i=1}^{N-j+1} C & \theta_{2i} \\ N & j-i & N-2i \\ \sum_{i=1}^{N-j+1} C & \theta_{2i-1} \end{bmatrix} & \text{if } N+1 < 2j \leq 2N, \end{cases} \quad (27)$$

$j = 2, 3, \dots, N$, θ_i are defined by the harmonics frequencies via (12).

The lemma is proved by defactorization of the discrete equation (8) shifted forward by $2N$ steps and introduction of the notations $\xi_i(k) = \psi(k-i+1)$ ($i = 1, 2, \dots, 2N$).

Remark 3. Due to the symmetry of combination, i.e., $\frac{j-i-1}{C} = \frac{N-i-j+1}{C}$ and $\frac{j-i}{N-2i} = \frac{N-i-j+1}{N-2i+1}$, $N-1$ element in the vector $\boldsymbol{\vartheta}$ are duplicated. Namely, it is seen from (26), (27) that

$$\boldsymbol{\vartheta} = \underbrace{[-1, \vartheta_2, \vartheta_3, \dots, \vartheta_{N+1}, \vartheta_N, \dots, \vartheta_2]^T}_{2N \text{ elements}}.$$

This fact will allow us to simplify the recalculation from the estimates of θ_i given by (20) (dependent from the frequencies) into the estimates of $\boldsymbol{\vartheta}$ used by the control law.

Remark 4. The function $\boldsymbol{\vartheta}(\boldsymbol{\theta})$ is linear and can be represented in the compact form

$$\boldsymbol{\vartheta} = \mathbf{m} + \mathbf{M}\boldsymbol{\theta}, \quad (28)$$

where

$$\mathbf{m} = \begin{bmatrix} \bar{\mathbf{m}}_1 \\ \bar{\mathbf{m}}_2 \\ \vdots \\ \bar{\mathbf{m}}_N \end{bmatrix} \in \mathbb{R}^{2N}, \mathbf{M} = \begin{bmatrix} \bar{\mathbf{M}}_1^T \\ \bar{\mathbf{M}}_2^T \\ \vdots \\ \bar{\mathbf{M}}_N^T \end{bmatrix} \in \mathbb{R}^{2N \times N} \quad (29)$$

are the vector and matrix given by

$$\bar{\mathbf{m}}_j = \begin{bmatrix} j-1 \\ -C \\ N \\ 0 \end{bmatrix} \text{ and } \bar{\mathbf{M}}_j^T = \begin{cases} \begin{bmatrix} \sum_{i=1}^{j-1} C & j-i-1 & \zeta_{2i}^T \\ N-2i & C & \zeta_{2i-1}^T \end{bmatrix} & \text{if } 2j \leq N+1, \\ \begin{bmatrix} \sum_{i=1}^{N-j+1} C & j-i-1 & \zeta_{2i}^T \\ N-2i & C & \zeta_{2i-1}^T \end{bmatrix} & \text{if } N+1 < 2j \leq 2N, \end{cases} \quad (30)$$

respectively, $j = 1, 2, \dots, N$, $\zeta_k = [0, \dots, 0, 1, 0, \dots, 0]^T$ ($k = 1, 2, \dots, N$) is the N -th dimensional coordinate vector.

Since the TV parameter $\psi(k)$ is unknown, the state ξ of the filter (25) is not measurable. However, it can be recovered via the observer recently proposed in [16].

Lemma 3. The observer

$$\hat{\xi}(k) = \zeta(k) + \mathbf{I} \frac{x(k-1)x(k)}{x^2(k-1) + \sigma^2(k-1)}, \quad (31)$$

$$\begin{aligned} \zeta(k+1) &= \mathbf{G}\zeta(k) + \mathbf{G}\mathbf{I} \frac{x(k-1)x(k)}{x^2(k-1) + \sigma^2(k-1)} - \\ &\quad - \mathbf{I} \frac{x(k)}{x^2(k) + \sigma^2(k)} u(k), \end{aligned} \quad (32)$$

where $\hat{\xi}$ is the estimate of ξ ,

$$\sigma(k) = \begin{cases} 0 & \text{if } |x(k)| \geq x_0, \\ \sigma_0 & \text{otherwise,} \end{cases} \quad (33)$$

$x_0, \sigma_0 > 0$ are constant design parameters, provides the following properties:

1. For any initial conditions $\xi(0), \zeta_j(0)$ the error $\epsilon(k) \triangleq \xi(k) - \hat{\xi}(k)$ satisfies the equality

$$\epsilon(k) = \mathbf{G}^k \epsilon(0) + (\mathbf{z}\mathbf{I} - \mathbf{G})^{-1} \mathbf{I} \left[\frac{\sigma^2(k)}{x^2(k) + \sigma^2(k)} \psi(k) \right]. \quad (34)$$

2. The norm $\|\epsilon(k)\|_\infty$ is bounded and can be reduced by reduction of the gain σ_0 and/or x_0 .

The equality (34) is proved by calculation of the next step value $\epsilon(k+1) = \xi(k+1) - \hat{\xi}(k+1)$ in view of (25), (31), (32), and (2). The boundedness of $\|\epsilon(k)\|_\infty$ is followed from (34) and the boundedness of the function

$$\frac{\sigma^2(k)}{x^2(k) + \sigma^2(k)} \psi(k).$$

Applying the result of Lemma 3, we represent the TV plant parameter in the form

$$\psi(k) = \boldsymbol{\vartheta}^T \hat{\xi}(k) + \boldsymbol{\vartheta}^T \epsilon(k). \quad (35)$$

Now, using the parameterization (35), in the next section we propose the adaptive controller.

Control law

Replacing (35) in the plant equation (2), we obtain the expression suitable for controller design:

$$x(k+1) = \hat{\boldsymbol{\theta}}^T \hat{\boldsymbol{\xi}}(k)x(k) + u(k) + f(k), \quad (36)$$

where $f(k) = \hat{\boldsymbol{\theta}}^T \boldsymbol{\epsilon}(k)x(k)$ is considered as an additive disturbance satisfying the property defined by the following lemma.

Lemma 4. $f(k) \in \mathcal{L}_\infty \forall k \geq 0$.

Proof. Using the notion of $\mathcal{L}_\infty$ -norms of functions in view of (34), we have

$$\begin{aligned} \|f(k)\|_\infty &= \left\| \hat{\boldsymbol{\theta}}^T \left(\mathbf{G}^k \boldsymbol{\epsilon}(0) + (\mathbf{z}\mathbf{I} - \mathbf{G})^{-1} \mathbf{I} \left[\frac{\sigma^2(k)}{x^2(k) + \sigma^2(k)} \psi(k) \right] \right) x(k) \right\|_\infty \\ &\leq \|\hat{\boldsymbol{\theta}}\|_\infty \left(\|\mathbf{G}^k \boldsymbol{\epsilon}(0)\|_\infty \|x(k)\|_\infty + \left\| \frac{\sigma^2(k)}{x^2(k) + \sigma^2(k)} \psi(k) \right\|_\infty \|x(k)\|_\infty \right). \end{aligned}$$

Here, we keep in mind that $\|(\mathbf{z}\mathbf{I} - \mathbf{G})^{-1} \mathbf{I}\|_\infty = 1$ and

$$\left\| \frac{\sigma^2(k)}{x^2(k) + \sigma^2(k)} \psi(k) \right\|_\infty \leq \frac{\sigma^2(k)}{\sqrt{x^2(k) + \sigma^2(k)}} \|\psi(k)\|_\infty.$$

Since the matrix $\mathbf{G}$ is nilpotent and $\mathbf{G}^{2N} = \mathbf{O}_{2N \times 2N}$, while $\|x(k)\|_\infty < \infty$ for all $k = 0, 1, \dots, 2N$, then

$$\|f(k)\|_\infty \leq \frac{\sigma^2(k)}{\sqrt{x^2(k) + \sigma^2(k)}} \|\hat{\boldsymbol{\theta}}\|_\infty \|\psi(k)\|_\infty \quad \forall k \geq 2N. \quad (37)$$

As a result, $\|f(k)\|_\infty$ is bounded $\forall k \geq 0$. This completes the proof.

Using (36), we select the adjustable control law in the form

$$u(k) = -\hat{\boldsymbol{\theta}}^T(k) \hat{\boldsymbol{\xi}}(k)x(k) - \lambda \varepsilon(k) + x_m(k+1), \quad (38)$$

where $\lambda \in \mathbb{R}$ is a constant design parameter such that $|\lambda| < 1$, $\hat{\boldsymbol{\theta}} \in \mathbb{R}^{2N}$ is the estimate of $\boldsymbol{\theta}$ given by replacement of $\hat{\theta}_i$ ($i = 1, 2, \dots, N$) instead of θ in (28):

$$\hat{\boldsymbol{\theta}} = \mathbf{m} + \mathbf{M}\hat{\boldsymbol{\theta}}. \quad (39)$$

In the later, the elements of $\hat{\boldsymbol{\theta}}$ are defined by the estimated frequencies of harmonics via (20) and generated by the adaptation algorithm (13) or (18).

Replacing (38) in (36) in view of (28) and (39), we obtain the error model

$$\varepsilon(k+1) = \lambda \varepsilon(k) + \hat{\boldsymbol{\theta}}^T(k) \mathbf{M}^T \hat{\boldsymbol{\xi}}(k)x(k) - f(k), \quad (40)$$

where $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$ is the identification error.

Now, using the error model (40) we are in position to formulate out main result.

Proposition 3. Under Assumption 1, the adaptive controller consisting of the adjustable control law (38), observer (31)–(33), identification algorithm (13) or (18), and the recalculation rule (29), (30), (39) being applied to the plant (2) ensures the following properties:

- P3.1 the boundedness of all the closed-loop signals in the system;
- P3.2 the objectives O1 and O2 according to the limits (22), (23);

— P3.3 $\tilde{\boldsymbol{\theta}}(k) = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(k)$ and $\omega_i - \hat{\omega}_i(k)$ tend to zero exponentially fast, and for the system closed by the identifier (18) the rate of the convergence $\tilde{\boldsymbol{\theta}}(k) \rightarrow 0$.

Proof. Let us rewrite plant model (2) with the parameterization (24) (see Lemma 2) and recalculation formula (28) in the following form:

$$\bar{y} = \boldsymbol{\theta}^T \bar{\boldsymbol{\phi}}, \quad (41)$$

where $\bar{y} = x(k) - u(k-1) - \mathbf{m}^T \boldsymbol{\xi}(k-1)x(k-1)$, $\bar{\boldsymbol{\phi}} = \mathbf{M}^T \boldsymbol{\xi}(k)x(k)$. Using these notations and the notation of $f(k)$, the model (40) can be represented as

$$\varepsilon(k+1) = \lambda \varepsilon(k) - \bar{e}(k) - \hat{\boldsymbol{\theta}}^T(k) \mathbf{M}^T \boldsymbol{\epsilon}(k)x(k), \quad (42)$$

where $\bar{e} \triangleq \bar{y} - \hat{\boldsymbol{\theta}}^T \bar{\boldsymbol{\phi}} = \tilde{\boldsymbol{\theta}}^T \bar{\boldsymbol{\phi}}$ is the identification error.

Using the arguments of Lemma 4 we can see that the signal $\hat{\boldsymbol{\theta}}^T \mathbf{M}^T \boldsymbol{\epsilon} x$ in (42) is bounded and satisfies the inequality

$$\begin{aligned} \|\hat{\boldsymbol{\theta}}^T \mathbf{M}^T \boldsymbol{\epsilon} x(k)\|_\infty &\leq \frac{\sigma^2(k)}{\sqrt{x^2(k) + \sigma^2(k)}} \|\hat{\boldsymbol{\theta}}^T \mathbf{M}^T\|_\infty \|\psi(k)\|_\infty \\ &\quad \forall k \geq 2N, \end{aligned} \quad (43)$$

where $\sigma(k)$ is given by (33).

For the regression (9), in accordance with Properties P1.1 and P1.2 of Propositions 1 and 2, both estimation schemes (13) and (18) ensure that $e(k) = y(k) - \hat{\boldsymbol{\theta}}(k)^T \boldsymbol{\phi}(k) = \tilde{\boldsymbol{\theta}}^T(k) \boldsymbol{\phi}(k)$, $\hat{\boldsymbol{\theta}}(k+1) - \hat{\boldsymbol{\theta}}(k) \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $e(k)$, $\hat{\boldsymbol{\theta}}(k+1) - \hat{\boldsymbol{\theta}}(k) \rightarrow 0$ as $k \rightarrow \infty$. Since the regressions (9) and (41) representing the same plant model have the same structure, then the estimators (13) and (18) additionally ensure that $\bar{e}(k) = \hat{\boldsymbol{\theta}}^T(k) \bar{\boldsymbol{\phi}}(k) \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $\bar{e}(k) \rightarrow 0$ as $k \rightarrow \infty$.

Since according to Propositions 1 and 2, $\hat{\boldsymbol{\theta}}(k) \in \mathcal{L}_\infty$ and, as shown above, the inequality (43) holds, then $\varepsilon(k) \in \mathcal{L}_\infty$. As a result, $x(k)$, $u(k) \in \mathcal{L}_\infty$. This completes the proof of Property P3.1.

If $|x(k)| \geq x_0$ for all $k \geq N_0$ ($N_0 \in \mathcal{N}$), then $\sigma(k) = 0$, $\boldsymbol{\epsilon}(k) = \boldsymbol{\xi}(k) - \hat{\boldsymbol{\xi}}(k) \rightarrow 0$ as $k \rightarrow \infty$. Therefore, $\varepsilon(k) \rightarrow 0$ as $k \rightarrow \infty$ according to (42). Otherwise, $\boldsymbol{\epsilon}(k)$, $f(k) \in \mathcal{L}_\infty$ according to (23). As seen from (37), Δ can be reduced by decreasing $\sigma(\sigma_0)$. This proves Property P3.2.

To prove Property P3.3, we take into account the equivalence of the regressors $\boldsymbol{\phi}(k)$ and $\bar{\boldsymbol{\phi}}$ for $x(k) \neq 0$. Since the generator (25), (24) is not redundant due to the knowledge of the number of harmonics N , $\boldsymbol{\xi}(k) \in \mathcal{PE}$. As a result, $\boldsymbol{\phi}$, $\bar{\boldsymbol{\phi}} \in \mathcal{PE}$ for $x(k) \neq 0$. Therefore, Property P3.3 is provided by the estimators (13) and (18) according to Propositions 1 and 2. This completes the proof of the proposition.

Now concluding the main result, we make the following remarks:

Remark 5.

- R5.1. As seen from the proof of Proposition 3, the problem objectives O1 and O2 do not require exact identification of the harmonics frequencies ω_i , however require convergence $\bar{e}(t) \rightarrow 0$. At the same time, since the exact number of harmonics is known, then the regressor $\boldsymbol{\phi} \in \mathcal{PE}$, and the parametric errors $\tilde{\boldsymbol{\theta}}(t)$ as well as the frequencies estimation errors converge to zero exponentially fast. As a result, the control error

$\varepsilon(t)$ converge to zero according to objective O1 or to the residual set according to objective O2 exponentially fast;

- R5.2. As seen from the structure of the plant (2), if the state $x(k)$ crosses zero, then the state $\xi(k)$ of the TV parameter model (25), (24) is no longer to be observable. In order to overcome this problem, the threshold x_0 is suitably involved into the observer. At the same time, in contrast to the method of direct adaptive control recently reported in [16], the proposed algorithm of frequencies estimation is independent from the value of $x(k)$ and the property of observability.

Simulation results

Consider the plant (2) with the uncertain time-varying parameter $\psi(k) = a_1 \sin(\omega_1 kT + \varphi_1) + a_2 \sin(\omega_2 kT + \varphi_2)$, in which $a_1, a_2, \omega_1, \omega_2, \varphi_1$, and φ_2 are unknown constants set to the following values: $a_1 = 0.2$; $a_2 = 0.3$; $\omega_1 = 2$; $\omega_2 = 3$; $\varphi_1 = 3$; and $\varphi_2 = 4$. The interval of discretization T is set to 0.1 (sec).

The problem is twofold:

- calculate the estimates $\hat{\omega}_i(k)$ ($i = 1, 2$) such that $\lim_{k \rightarrow \infty} (\omega_i - \hat{\omega}_i(k)) = 0$;

- design a control law ensuring the boundedness of all the closed-loop signals and driving the tracking error $\varepsilon(k) = x_m(k) - x(k)$ to zero, where $x_m(k)$ is generated by the reference model $x_m(k+1) = 2 + \sin(2kT)$.

In the simulation, we select the estimator with MRE

$$(18), \text{ for which we select the first-order filter } L(z) = \frac{0.3}{z - 0.7}.$$

For the TV parameter observer, we select

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{I} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$x_0 = 0.01$, and $\sigma_0 = 0.01$, while for the control law given by (38) we choose $\lambda = 0.1$.

All initial conditions in the scheme are set to zero.

The simulation results are presented in Fig. 1 and Fig. 2 and demonstrate the convergence of the frequencies estimates $\hat{\omega}_i$ ($i = 1, 2$) to the original values ω_i and, as a result, the convergence of the tracking error to zero. As shown by Fig. 2, the rate of estimation and, as a result, the rate of reference tracking can be increased by increasing the coefficient gamma.

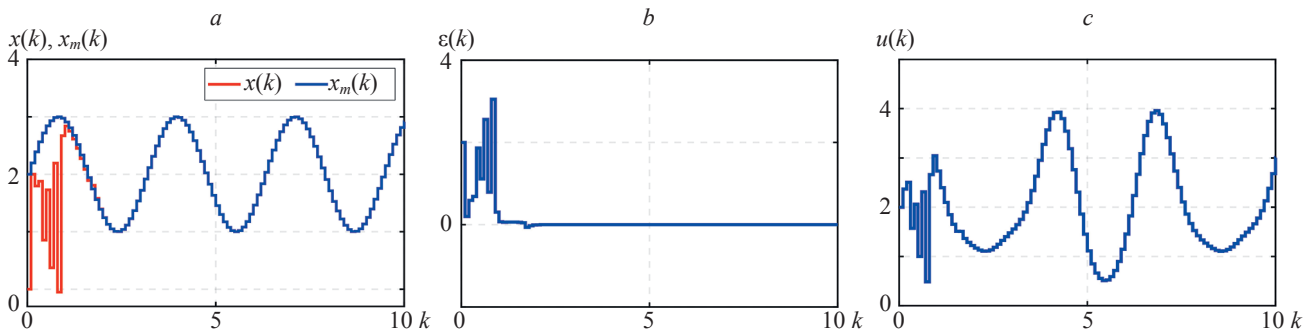


Fig. 1. Transients in the system closed by the estimator with MRE (18) with $\gamma = 100$. Transients of: the state $x(k)$ and the reference model output $x_m(k)$ (a); the tracking error $\varepsilon(k)$ (b); the control signal $u(k)$ (c)

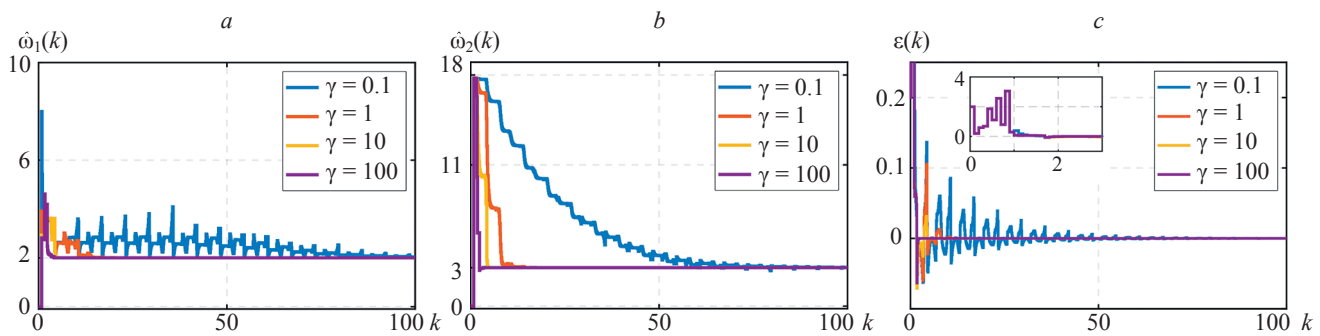


Fig. 2. Response frequency estimates $\hat{\omega}_1$ (a), $\hat{\omega}_2$ (b) and the tracking error $\varepsilon(k)$ (c) in the system closed by the estimator with MRE (18) for different γ

Conclusion

In the paper, two adaptive frequencies estimators for the first order linear discrete system with uncertain multisinusoidal TV parameter are designed. The first estimator is based on the gradient scheme of adaptation, while the second one uses the discrete version of the

adaptation algorithm with MRE. Then, the indirect (identification-based) adaptive control laws using these estimators are proposed, and their properties are analyzed.

The estimation algorithms together with the control laws are supposed to be developed toward the following axes:

- solution to the problem for extended class of plants [16]

$$\begin{aligned}x_i(k+1) &= \psi_i(k)x_i(k) + x_{i+1}(k), \\x_n(k+1) &= \psi_n(k)x_n(k) + u(k), \\y(k) &= x_1(k),\end{aligned}$$

where $i = 1, 2, \dots, n-1$, $x_j \in \mathbb{R}$ ($j = 1, 2, \dots, n$) are the elements of the state vector $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$ with the initial condition $x_j(0)$, $y \in \mathbb{R}$ is the plant output, $u(k) \in \mathbb{R}$ is the control signal, $\psi_j(k)$ are the unknown parameters given by the functions $\psi_j(k) = \sum_{i=1}^N a_{j,i} \sin(\omega_{j,i} kT + \phi_{j,i})$, with

unknown constants $a_{j,i}$, $\phi_{j,i}$ and $\omega_{j,i}$. The maximum numbers of harmonics N_j are assumed known.

— solution to the problem for continuous time systems of the form

$$\dot{\mathbf{x}} = \psi(t)\mathbf{x} + \mathbf{u},$$

where $\psi(t) = \sum_{i=1}^N a_i \sin(\omega_i kT + \phi_i)$ is the uncertain TV parameter with unknown constant a , ω and ϕ .

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