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Geometric modeling and compensation of cutting tool positioning errors for eliminating protrusion in large-radius spherical surface machining

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Abstract
The production of optical components with a large radius of spherical surfaces requires exceptionally high surface profile accuracy. Minor deviations in the positioning of the cutting tool caused by factors, such as mechanical backlash, thermal deformation, and incorrect tool positioning, can result in dimensional errors of the machined surface, particularly in the form of protrusions that indicate processing defects. Despite a wide range of studies focused on tool wear and general machining errors, insufficient attention has been given to the geometric modeling and correction of defects caused by tool positioning errors. This study presents a comprehensive approach to geometrically modeling the impact of cutting tool positioning errors on the machined surface profile. A mathematical model has been developed to model the interaction between the tool and the spherical surface, enabling precise estimation of the radial machining error. Based on these data, a new error compensation method is proposed, allowing for the correction of errors by modifying the tool movement trajectory. The proposed model accurately predicts the formation and characteristics of protrusions resulting from tool displacement during the machining of spherical surfaces with a large radius. Implementation of the compensation method significantly reduces the defect rate, improves geometric accuracy, and decreases the need for additional processing. Addressing defects caused by positioning errors enables the proposal of a new method that has not previously been considered in precision machining research. The proposed model and tool positioning error compensation method offer an effective and practical solution for improving the surface profile accuracy of optical components, thereby enhancing the precision and efficiency of manufacturing processes. The proposed method contributes to the advancement of high-precision optical component manufacturing with minimal post-processing costs, providing a novel approach in the fields of instrument engineering and precision mechanical engineering.

Keywords
machining errors, tool wear, precision manufacturing, concave surface milling, calibration techniques, mathematical modeling

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Геометрическое моделирование и компенсация ошибок позиционирования режущего инструмента для устранения выступов при обработке оптических поверхностей с большим радиусом сферы

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Аннотация

Введение. Производство оптических компонентов с большими радиусами сферических поверхностей требует исключительно высокой точности профиля поверхности. Незначительные отклонения в позиционировании режущего инструмента, вызванные такими факторами, как механический люфт, тепловая деформация и ошибочное позиционирование инструмента, могут привести к ошибке в размере обработанной поверхности — в частности, выступам, которые свидетельствуют об ошибке обработки. Несмотря на широкий спектр исследований, посвященных износу инструмента и общим ошибкам обработки, недостаточно внимания уделено геометрическому моделированию и коррекции дефектов, вызванных ошибками позиционирования инструмента. **Метод.** Представлен комплексный подход к геометрическому моделированию влияния ошибок позиционирования режущего инструмента на профиль обработанной поверхности. Разработана математическая модель, имитирующая взаимодействие инструмента и сферической поверхности, что позволяет точно оценить ошибку радиального размера обработки. На основе этих данных предложен новый метод компенсации ошибки, позволяющий корректировать ошибки путем изменения траектории перемещения инструмента. **Основные результаты.** Предложенная модель с высокой точностью предсказывает формирование и характеристики выступов, возникающих вследствие смещения инструмента при обработке сферических поверхностей с большим радиусом. Внедрение метода компенсации существенно снижает количество брака, улучшая геометрическую точность и уменьшая потребность в дополнительной обработке. **Обсуждение.** Рассмотрение дефектов, вызванных ошибками позиционирования, позволяет предложить новый метод, ранее не рассмотренный в исследованиях по точной обработке. Предлагаемая модель и метод компенсации погрешности позиционирования инструмента демонстрируют эффективное и практическое решение для повышения точности профиля оптических компонентов, что способствует увеличению точности и эффективности производственных процессов. Разработанный метод вносит вклад в развитие высокоточного производства оптических изделий с минимальными затратами на постобработку, обеспечивая новый подход в области приборостроения и прецизионного машиностроения.

Ключевые слова

погрешности обработки, износ инструмента, высокоточное производство, фрезерование вогнутых поверхностей, методы калибровки, математическое моделирование

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Introduction

The instrumentation and precision manufacturing industries have witnessed remarkable advancements in recent years, primarily driven by the increasing demand for ultra-high accuracy components in fields, such as optics, aerospace, and microelectronics. Within these domains, particularly in the fabrication of optical components, geometric precision and surface integrity, are paramount. Optical elements, such as concave mirrors, spherical lenses, and freeform surfaces, must exhibit strict dimensional fidelity and surface smoothness to achieve their intended optical performance. One of the persistent and critical challenges in achieving such accuracy lies in the occurrence of cutting tool positioning errors which introduce localized surface anomalies such as protrusions and profile distortions, especially in the machining of large-radius spherical surfaces.

Cutting tool positioning error refers to the deviation of the actual tool location from its programmed position during the machining process. These deviations can arise from a variety of sources, including mechanical backlash, thermal deformation, encoder inaccuracies, servo lag, tool-holder deflection, or misalignment during setup. In multi-axis Computer Numerical Control (CNC) machining environments, such errors become increasingly impactful, particularly when the contact geometry between the tool and workpiece is sensitive to small variations in tool orientation or position. This sensitivity is magnified in large-radius spherical surfaces where the surface curvature is subtle — thus any slight misalignment of the cutting edge can result in geometrically significant protrusions or irregularities that degrade the optical quality of the component.

In modern manufacturing systems, closed-loop machine tools and precision equipment are often employed to mitigate such errors. However, even high-end imported machinery suffers from degradation over time, limited setup flexibility, and environmental influences such as thermal fluctuations, which further contribute to mispositioning. These conditions are exacerbated in complex geometries like concave or freeform surfaces where the spatial tool–surface interaction is nontrivial and continuously changing. Therefore, accurately modeling, detecting, and compensating for tool positioning errors becomes crucial for ensuring the desired surface profile and minimizing costly rework or post-processing.

Numerous studies have addressed the general problem of machining errors, surface roughness, and tool wear in freeform and precision surface manufacturing. For instance, in [1] introduced a surface-shape-based method for finishing freeform surfaces, optimizing local finishing performance but assuming ideal tool positioning. Likewise, in [2] modeled the topology of freeform surfaces generated by ball-end milling, providing critical insight into surface generation mechanics but not explicitly accounting for spatial tool mispositioning. The sensitivity of optical components to surface errors was further emphasized in [3], which demonstrated the design and fabrication of concave lens arrays on aspheric curved surfaces — highlighting the need for high geometric fidelity throughout the process.

In [4] focused on the surface roughness variations during concave surface milling of AISI 420 steel, exploring how process parameters affect surface texture, though again under the assumption of ideal tool trajectory. A more direct link to positioning control is found in [5] which proposed a double-point contact tool positioning method using a barrel cutter for concave surfaces, thereby enhancing surface conformity and reducing form error. Similarly, in [6] investigated the influence of tool path strategies and cutting parameters on tool deflection and cutting forces — factors that directly impact the spatial accuracy of tool positioning.

Advances in CNC programming and specialized tooling have also contributed to reducing form errors. Research [7, 8] introduced precision machining methods for concave-arc and cone-end milling cutters respectively, offering improved tool profiles that can reduce sensitivity to positional deviations. Additionally, in [9] explored surface micro-textures to enhance anti-adhesion properties, indirectly contributing to better tool stability and reduced deviation during dry cutting conditions. In [10] presented CNC form milling for concave-convex arc-line gears, underscoring the precision demands in gear tooth profile machining.

From a modeling perspective, mathematical approaches have been used to predict and correct machining deviations. In [11] presented a model for machining internal double-arc spiral bevel gears using finger milling cutters, integrating geometric and kinematic considerations relevant to positioning accuracy. In [12] laid out foundational knowledge in optics fabrication and precision machining methodologies, establishing key metrics for surface quality and geometric error evaluation. Furthermore, in [13] examined geometrical errors in surfaces produced using concave and convex profile tools, highlighting the need for precise tool placement during milling. Research [14] provided a comprehensive review of polishing techniques for optical parts, emphasizing the need for minimal surface error prior to finishing. More recently, in [15] investigated the influence of tool structure and cutting orientation on force generation, offering insight into the positional stability of double-arc milling cutters under dynamic load conditions.

Despite these advances, there remains a significant gap in the literature regarding the explicit modeling and compensation of tool positioning errors as a source of localized protrusions or profile distortions, especially in the context of large-radius spherical surface machining. Traditional error compensation methods tend to focus on general tool wear, thermal effects, or vibration-related deviations, and often assume a predictable, uniform error distribution. However, positioning errors are frequently non-uniform and manifest as localized surface features which are difficult to detect in-line and are even harder to remove through polishing or re-machining without compromising the global geometry.

To bridge this gap, the present study proposes a geometric modeling framework and compensation strategy specifically targeting surface deviations caused by cutting tool positioning errors. The focus is on the manufacturing of optical parts with large-radius spherical surfaces, where the tool-surface interaction is highly susceptible to positional misalignments. A detailed mathematical model is developed to analyze how deviations in tool position translate into profile errors, particularly protrusions that are often observed in post-process inspection of high-precision optical components.

The model incorporates parameters, such as tool geometry, intended tool path, surface curvature, and local tool orientation. Furthermore, a compensation strategy is proposed that can either adjust the tool path in the Computer-Aided Manufacturing software prior to machining or guide post-process corrective actions to eliminate the resulting surface defects.

Experimental observation of protrusion formation

Precision machining of large radius spherical surfaces requires strict control over cutting tool positioning, as even minor deviations can lead to significant geometric defects. A critical issue observed in the milling of concave optical components, particularly on the LOH300 machine, is the formation of a central protrusion, a defect directly linked to tool alignment errors and path inaccuracies. This protrusion degrades both the geometric accuracy and optical functionality of the machined surfaces.

In the standard setup, the workpiece rotates on a table while the inclined milling cutter feeds vertically, with longitudinal adjustments refining the cutting path. This configuration enables high precision spherical concavities, but its success hinges on exact tool positioning. Deviations in the cutter inclination angle, vertical descent, or longitudinal displacement disrupt the intended tool workpiece interaction, often resulting in a residual central protrusion. Such errors are exacerbated in large radius machining where slight misalignments produce amplified surface distortion.

To visualize the setup, consider the schematic shown in Fig. 1, which illustrates the spatial arrangement of the milling cutter and the rotating workpiece. As shown, the inclined cutter rotating edge intersects with the workpiece surface to remove material, thereby generating the concave spherical profile.

To better understand the material removal process, a cross-sectional analysis of the cutter and workpiece interaction is presented in Fig. 2. The cutting edge of the milling tool traces a spatial path that forms a segment of a toroidal surface. This toroidal surface intersects with the spherical volume of the rotating workpiece. Material removal occurs at the intersection point, generating a concave surface whose curvature is a function of both cutter geometry and motion parameters.

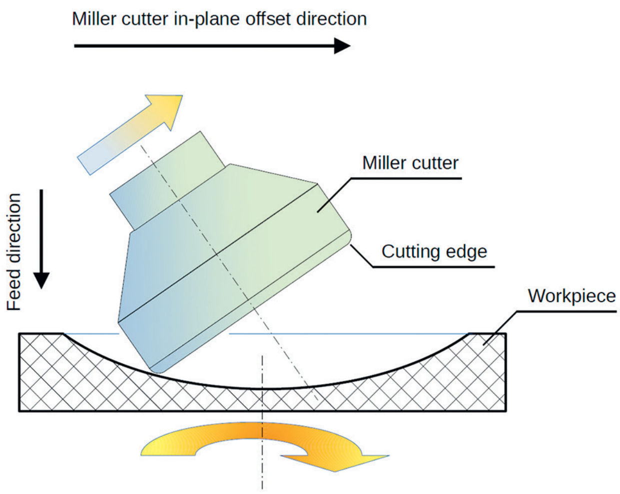

Fig. 1. Schematic of the milling setup

The cutter is capable of longitudinal and angular movement relative to the rotation axis of the workpiece. Errors in these movements can result in undercutting, manifesting as a protrusion at the center of the concave surface.

The physical manifestation of this defect is shown in Fig. 3. A clear central protrusion appears at the center of the machined spherical surface, resulting from misalignment in the cutter path relative to the workpiece axis. This geometrical error distorts the intended concave profile and is especially problematic in precision applications like optical components.

After identifying the source of the error, corrective measures were applied to adjust the cutter inclination angle and ensure proper longitudinal feed displacement. The result of these adjustments is presented in Fig. 4.

As seen in Fig. 4, the protrusion has been completely removed. The surface now conforms to the desired spherical geometry, confirming that precise tool alignment and feed optimization are essential for defect-free machining.

This real-world observation provides context for the geometric behavior illustrated in the following schematic, which conceptually models the cutter misalignment and its consequence on surface geometry.

Geometric modeling of cutter misalignment effects

The phenomenon of central protrusion is further analyzed through geometric modeling. Fig. 5 depicts how cutter misalignment during the milling process results in a protrusion forming at the center of the machined surface.

To mathematically describe the protrusion formation, the motion of the cutter edge relative to the workpiece is modeled using hypocycloidal geometry which can be described by the following parametric equation:

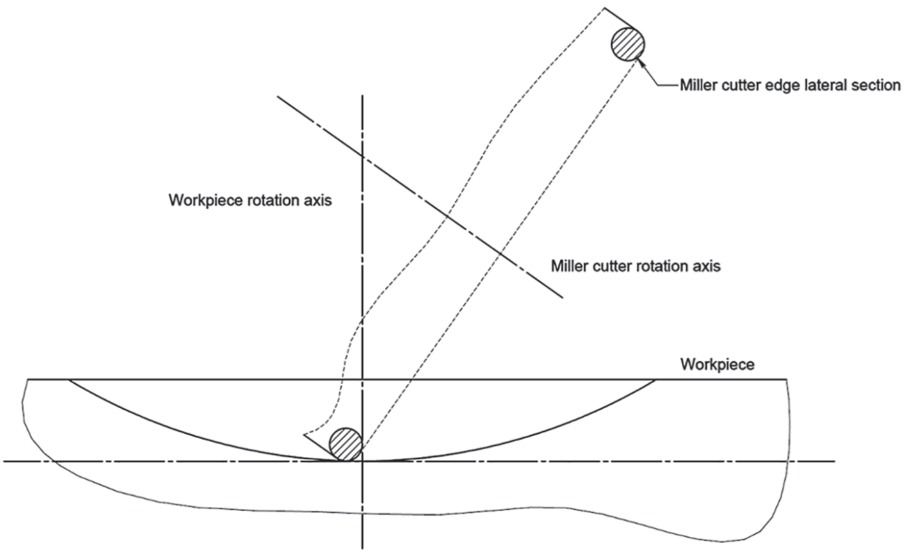

Fig. 2. Cross-section of the milling cutter and workpiece during processing

$$\begin{cases} x = (R_L - R_S)\cos\beta + R_S\cos\dfrac{R_L - R_S}{R_S}\beta \\ y = (R_L - R_S)\sin\beta - R_S\sin\dfrac{R_L - R_S}{R_S}\beta \end{cases}, \qquad (1)$$

where R_L is the radius of the larger circle, R_S is the radius of the smaller circle, and β is the rotation angle of the smaller circle.

When a circle with radius R_S rolls along the inner side of a circle with radius R_L by an angle β, point A, located on the circumference of the smaller circle, moves to point A', according to the equation (1) the coordinates change as the following equation:

$$A(\beta) = \left((R_L - R_S)\cos\beta + R_S\cos\frac{R_L - R_S}{R_S}\beta,\; (R_L - R_S)\sin\beta - R_S\sin\frac{R_L - R_S}{R_S}\beta\right). \qquad (2)$$

Fig. 3. Machined part before correction — central protrusion clearly visible

Fig. 4. Machined part after proper tool alignment — protrusion successfully removed

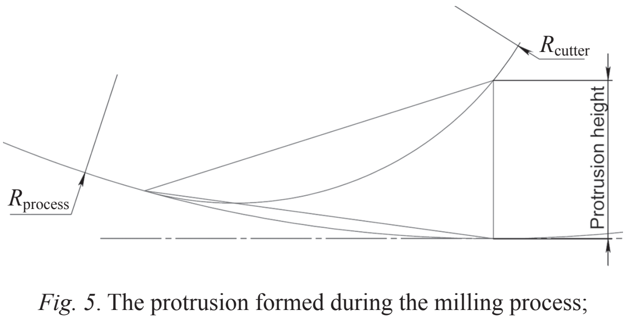

Fig. 5. The protrusion formed during the milling process; $R_{process}$ is desired radius of the machined spherical surface; R_{cutter} is the radius of the milling cutter tip used for machining

This motion is conceptually depicted in Fig. 6, where the smaller circle rolls inside the larger circle and traces a path via a fixed point on its circumference.

The central protrusion can also be analyzed geometrically as shown in Fig. 7. A larger circle with R_L and center at point O represents the reference spherical surface. The smaller circle rolls along the inner circumference from left to right. A vertical line OB intersects the lower point of the larger circle. Point A marked on the smaller circle rolls and eventually coincides with point B on line OB. The straight-line AO connecting point A to the center O change its length during this motion, ranging from $R_L - R_S$ to R_L.

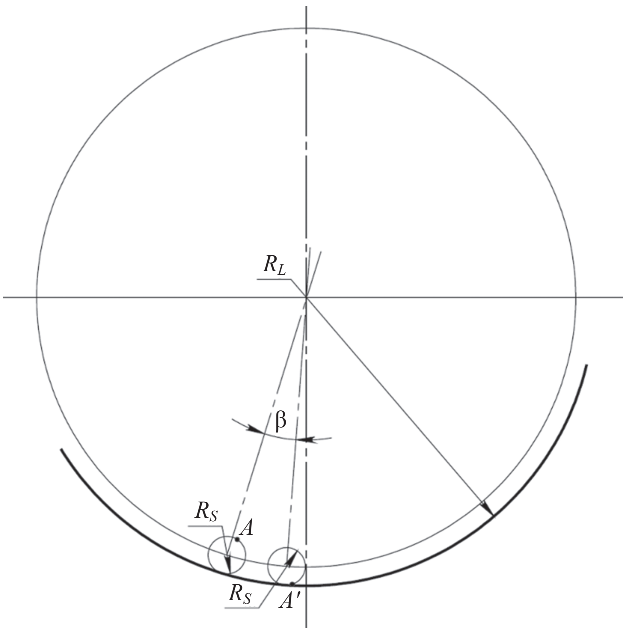

Fig. 6. Rolling of a smaller-radius circle along the inner circumference of a larger-radius circle

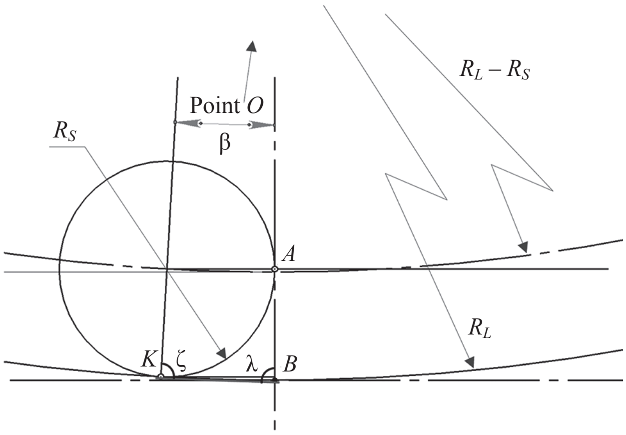

Fig. 7. The rolling of the milling cutter onto the workpiece (point O is outside the drawing); K is a point of tangency between the milling cutter and the machined surface profile; ζ is the angle formed at point K between the milling cutter radius and the machined surface; λ is angle at point B between the machined surface profile and the horizontal reference (axis)

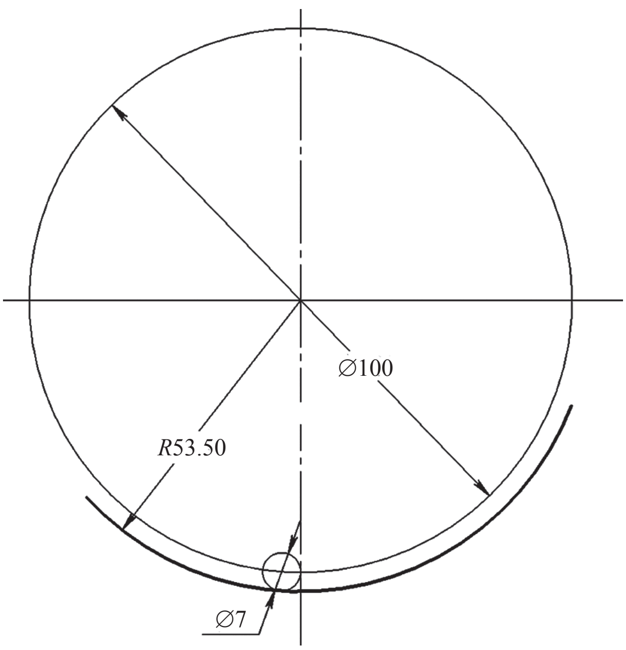

Fig. 8. Cross-section of the milling cutter and the workpiece (diameter 100 mm refers to the cutting radius center point)

The length of the line segment AO can be calculated as a function of angle β:

$$AO(\beta) = \sqrt{(x_A - x_O)^2 + (y_A - y_O)^2}, \qquad (3)$$

where (x_O, y_O) is the coordinate of point O, which are both zero; (x_A, y_A) is the coordinate of point, which were determined in equation (2). By substituting the values from equation (2) into equation (3), the full expression becomes:

$$AO(\beta) = \sqrt{\left((R_L - R_S)\cos\beta + R_S\cos\frac{R_L - R_S}{R_S}\beta\right)^2 + \left((R_L - R_S)\sin\beta - R_S\sin\frac{R_L - R_S}{R_S}\beta\right)^2}. \qquad (4)$$

Geometric analysis of tool-surface interaction in spherical machining

To validate the analytical model and gain visual insight into the tool movement and material engagement, a computation study was conducted using numerical modeling. The objective of the model was to replicate the real-world conditions of the machining process and examine the interaction between the cutter and the workpiece, especially in the region near the axis of rotation.

The analysis modeled a two-dimensional cross-section of the cutter and the workpiece, capturing the exact geometry of the toroidal tool and the spherical cavity. Parameters used in the model included:

— Cutter radius R_S = 3.5 mm;
— Workpiece radius R_L = 53.5 mm.

This setup is visualized in Fig. 8.

A key part of the modeling involved determining how far the cutter needed to move to ensure full coverage of the spherical surface. This was analyzed by constructing an arc that represented the necessary rolling distance of the cutter. Fig. 9 shows this construction which defines the chord length and the corresponding arc required for the tool edge to move from the periphery to the central region.

Using this setup, the angle β corresponding to the arc length was measured to be 6°09' (for substitution into the hypocycloid model 0.1073 rad). Substituting angle value into equation (4), precise distance from the cutter edge to the workpiece center during operation is computed

Evaluation of protrusion formation based on tool path error analysis

The computational analysis results validated the theoretical prediction that a hypocycloidal cutter trajectory may produce a central protrusion if the longitudinal displacement is not properly aligned. Specifically, the computed value of $AO(\beta)$, using equation (4) with β = 0.1073 rad, yielded a distance of 50.871 mm. This result closely approximates the target spherical radius R_L = 53.5 mm, thereby demonstrating the high accuracy and practical relevance of the analytical model in the context of precision machining.

The close agreement between model-based results and analytical predictions confirms the suitability of hypocycloidal trajectory modeling for analyzing and optimizing spherical surface machining. In particular, the visual outputs of the computational model clearly illustrated that insufficient longitudinal feed, tool inclination error, or tool positioning inaccuracies can result in undercutting or surface protrusions at the center of the machined region. Such deviations highlight the sensitivity of the process to even minor misalignments in the cutter path.

Beyond validating the analytical formulation, the computational modeling process also provided valuable insights into machining process optimization. It was shown that increasing the longitudinal feed displacement or adjusting the cutter inclination angle can ensure full engagement between the tool and the spherical surface. Moreover, minimizing tool positioning errors — both in the feed direction and angular alignment — is essential

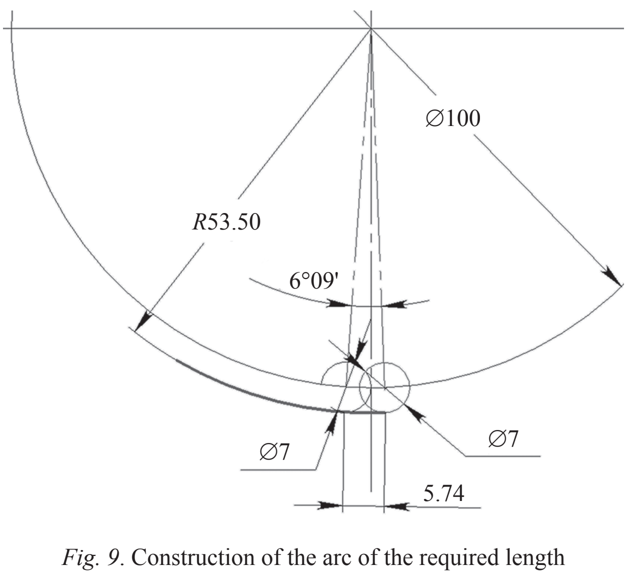

Fig. 9. Construction of the arc of the required length

to maintain geometric accuracy and prevent surface defects. These measures help improve surface finish, reduce dimensional deviations, and minimize the need for corrective rework.

These findings have direct implications for high-precision machining applications, particularly in areas such as optical component manufacturing where sub-micron accuracy and flawless surface integrity are critical. Incorporating the hypocycloidal trajectory model into machining strategies enables accurate prediction and compensation for potential deviations caused by tool misalignment or positioning errors. The strong correlation between the analytical model and computed results confirms that this approach can significantly enhance machining reliability and accuracy, especially for components with large-radius spherical geometries.

Conclusion

This study presented a comprehensive investigation into the geometric modeling and compensation of cutting tool positioning errors in the machining of large-radius spherical surfaces. Starting from experimental observations of central protrusion defects, the work identified that minor misalignments in the tool inclination or longitudinal feed can lead to significant surface deviations, particularly in applications requiring sub-micron geometric fidelity such as optical components. A mathematical framework based on hypocycloidal geometry was developed to describe the cutter motion relative to the spherical surface. This model accurately predicted the formation and characteristics of protrusions and was supported through geometric constructions reflecting real-world tool-surface interaction. The consistency between analytical predictions and modeled tool paths confirmed the effectiveness of the proposed approach. Beyond theoretical formulation, the study demonstrated that adjusting tool inclination and feed displacement, guided by the geometric model, effectively eliminates protrusions and restores the intended surface geometry. These findings were reinforced by corrected machining outcomes, where the surface profile was restored after applying model-based adjustments. By addressing a specific and underexplored class of machining errors and localized defects caused by the cutter mispositioning, this research provides a robust predictive model and compensation strategy. The method is both practical and adaptable, offering a valuable approach for improving surface integrity, reducing post-processing and enhancing reliability in ultra-precision manufacturing environments.

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